ANALYTICAL MODEL OF THE DIGITAL ANTENNA ARRAY ON A BASIS OF FACE-SPLITTING MATRIXS PRODUCT

V. I. Slyusar

Scientific Centre of problems of protection from the precision weapon at Kiev institute of Forces Ground
Kiev, Andruschenko street, 4

Under consideration of multicoordinate digital antenna arrays (DAA) there is the problem of compact matrix record of the responses of reception channels. The known mathematical apparatus does not allow to use habitual for the perception structures of matrixes, describing directivity characteristics of the antenna elements and amplitude-frequency characteristics of the filters. For the solution of the given problem it is offered to operate with a special type of the product of matrixes, named by the author as "face-splitting". According to the definition, for p×g - matrix A and p×s- matrix B their face-splitting product $A \Box B$ is, that each i-element of row of matrix A is multiplied on the appropriate row of matrix B. Result of the face-splitting product $A \Box B$ is the p×gs- block-matrix $\left[a_{ij} \cdot B_i\right]$, where B_i – is row of matrix B corresponding to element a_{ij}.

As an example, the response of three-coordinate flat equidistant DAA of *R*×*R* elements can be written down through face-splitting product of matrixes as:

$$U = S^{*}(A \Box Q \Box V), \qquad (1)$$

where A – is the vector of complex amplitudes of signals of M sources; Q, V – is M×R – matrix of the directivity characteristics of secondary channels in azimuth and elevation angle planes:

$$Q = \begin{bmatrix} Q_1(x_1) & Q_2(x_1) & \cdots & Q_R(x_1) \\ Q_1(x_2) & Q_2(x_2) & \cdots & Q_R(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ Q_1(x_M) & Q_2(x_M) & \cdots & Q_R(x_M) \end{bmatrix},$$

$$V = \begin{bmatrix} V_1(y_1) & V_2(y_1) & \cdots & V_R(y_1) \\ V_1(y_2) & V_2(y_2) & \cdots & V_R(y_2) \\ \vdots & \vdots & \vdots & \vdots \\ V_1(y_M) & V_2(y_M) & \cdots & V_R(y_M) \end{bmatrix};$$

S – is M×T – matrix of meanings of amplitude-frequency characteristics of T filters at frequencies of M signals; U – is T×RR -block- matrix of complex voltages R×R of secondary DAA channels on an output of T frequency filters,

$$U = \left[\begin{array}{cccc|c} U_{111} & U_{111} & \cdots & U_{1R1} & \cdots \\ U_{211} & U_{221} & \cdots & U_{2R1} & \cdots \\ \vdots & \vdots & & \vdots & \vdots \\ U_{T11} & U_{T21} & \cdots & U_{TR1} & \cdots \end{array}\right.$$

$$\left.\begin{array}{c|cccc} \cdots & U_{11R} & U_{12R} & \cdots & U_{1RR} \\ \cdots & U_{21R} & U_{22R} & \cdots & U_{2RR} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & U_{T1R} & U_{T2R} & \cdots & U_{TRR} \end{array}\right]$$

and $U_{trn} = \sum_{m=1}^{M} S_t(w_m) \cdot Q_r(x_m) \cdot V_n(y_m)$.

Among the basic properties of the face-splitting product for matrixes with identical quantity of rows it can be specified the following:

$$(A \Box B) \Box C = A \Box (B \Box C),$$
$$(A + B) \Box C = A \Box C + B \Box C, \; A \Box B \neq B \Box A,$$
$$A \otimes (B \Box C) = (A \otimes B) \Box C.$$

However, the most specific is the display of the law of order inversion. In distinguish with usual matrix product, when $(A \cdot B)^T = B^T \cdot A^T$, in a case of face-splitting multiplication it is written down as:

$$(A \Box B)^T = A^T \blacksquare B^T,$$

where of $\blacksquare$ – is symbol of transposed face-splitting product (TFSP).

The essence of TFSP consists in multiplication of elements of the left matrix by columns of the right so, that for g×p – matrix $A = \left[a_{ij}\right]$ and s×p – matrix B, submitted as a the block – matrix of columns ($B = \left[B_j\right]$, j = 1,..., *p*), the equality is fair

$$A \blacksquare B = \left[a_{ij} \cdot B_j\right].$$

With the help of TFSP it is possible to obtain the alternate to considered above variant of analytical model of three-coordinate DAA

$$\tilde{U} = (Q^{*} \blacksquare V^{*} \blacksquare S^{*}) \cdot A. \qquad (2)$$

If A – is a vector of complex amplitudes, $\tilde{U}$ – a block-vector of voltages of the responses of DAA channels. At multidimensional measurements, when A becomes a matrix of meanings of complex

Proc. of the 2nd International Conference on Antenna Theory and Techniques, 20–22 May 1997, Kyiv, Ukraine

amplitudes in various moments of time, $\tilde{U}$ turns correspondingly into the block- matrix.

Comparing record (1) and (2), it is necessary to note, that the first of them gives more compact result, however the expression (2) is more preferable from the point of view of possibility of preservation on its basis of continuity of methods of processing of signals in DAA of any dimension.

In particular, using the method of the maximum likelihood, the estimation of parameters of *M* sources of signals of four-coordinate DAA it is possible to carry out, by minimization of functional, not distinguished by the form from used in one-coordinate case. Really it is possible to write down:

$$L = \{U - FA\} * \{U - FA\} = \min,$$

$F = Q^* \blacksquare V^* \blacksquare S^* \blacksquare Z^*$ – for the selection on all four coordinates (Z^* – is the matrix of the responses of M signals in Z range gates) or $F = Q^*$ – at measurements of only in one of angle planes.

The measuring procedure in four-coordinate variant is reduced to minimization of expression:

$$L = tr[G \cdot R], \quad \text{where} \quad G = F \cdot (F^* \cdot F)^{-1} \cdot F^*,$$

$$R = U \cdot U^*, \; F = Q^* \blacksquare V^* \blacksquare S^* \blacksquare Z^*.$$

Similarly it is possible to present an algorithm of suppression of active countermeasures, based on the ortogonalization. Leaning on the calculations for one-coordinate problem, the resulting vector of signals voltage after suppression of active countermeasures we shall write down as:

$$U_B = \tilde{G} \cdot U_{исх}, \text{ where, } G = 1 - F \cdot (F^* \cdot F)^{-1} \cdot F^*,$$

$$F = Q^* \blacksquare V^* \blacksquare S^* \blacksquare Z^*.$$

Here columns of matrixes, forming F, correspond to those meanings of coordinates, where there are the zero of the gear function of DAA.