ACCURACY OF LINEAR DIGITAL ANTENNA ARRAY AT JOINT ESTIMATION OF RANGE AND ANGULAR COORDINATE OF M SOURCES

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For the analysis of accuracy of linear digital antenna array (DAA) it is offered to use the matrix record of its response of a kind:

$$U = P \cdot A \;, \qquad (1)$$

where $P = S \blacksquare F$, $A = \begin{bmatrix} a_1 & a_2 & \cdots & a_M \end{bmatrix}^T$ – the vector of complex signals amplitudes,

$$S = \begin{bmatrix} S_1(z_1) & S_1(z_2) & \cdots & S_1(z_M) \\ S_2(z_1) & S_2(z_2) & \cdots & S_2(z_M) \\ \vdots & \vdots & \vdots & \vdots \\ S_T(z_1) & S_T(z_2) & \cdots & S_T(z_M) \end{bmatrix} -$$

$T \times M$ - matrix of the responses of T gates of range on M of signals,

$$F = \begin{bmatrix} F_1(x_1) & F_1(x_2) & \cdots & F_1(x_M) \\ F_2(x_1) & F_2(x_2) & \cdots & F_2(x_M) \\ \vdots & \vdots & \vdots & \vdots \\ F_R(x_1) & F_R(x_2) & \cdots & F_R(x_M) \end{bmatrix} -$$

$R \times M$-matrix of meanings of the directivity characteristics of R reception channels of DAA in directions of M sources ; $\blacksquare$ - symbol of transposed face-splitting matrixes product (is entered by the author [1]).

With the account of (1), on the basis of matrix Neudecker's derivative [2] an information Fisher's block-matrix, describing the accuracy of joint estimation of range and angular coordinate, is obtained:

$$I = \frac{1}{\sigma^2} \times$$

$$\times \begin{bmatrix} P^T \cdot P & \vdots & (A^* \otimes P^T) \cdot \dfrac{\partial P}{\partial Y} \\ \cdots\cdots\cdots & \vdots & \cdots\cdots\cdots \\ \left(\dfrac{\partial P}{\partial Y}\right)^T \cdot (A \otimes P) & \vdots & \left(\dfrac{\partial P}{\partial Y}\right)^T \cdot (AA^* \otimes 1_{RT}) \cdot \dfrac{\partial P}{\partial Y} \end{bmatrix},$$

where $\dfrac{\partial P}{\partial Y}$ – Neudecker's derivative of matrix P on vector Y, made of unknown estimations of range and angular coordinates of M sources; 1_{RT} – identity matrix of dimension $R \times T$; $\otimes$ – symbol of Kronecker- products of matrixes.

As in the considered case we are interested only in dispersions of components of vector Y, it is necessary to form only the right bottom block of matrix $H = I^{-1}$. According to a procedure of the block-matrix inversion, the necessary block of a matrix H will be written down as:

$$H_{22} = \left[C - B^* \cdot A^{-1} \cdot B\right]^{-1}, \qquad (2)$$

where $A = P^T \cdot P$, $B = (A^* \otimes P^T) \cdot \dfrac{\partial P}{\partial Y}$,

$$C = \left(\frac{\partial P}{\partial Y}\right)^T \cdot (AA^* \otimes 1_{RT}) \cdot \frac{\partial P}{\partial Y}.$$

It is essential, that for preservation of the dependence of dispersion of estimations of ranges and angular coordinates of M sources on differences of initial phases of their signals, it is necessary in (2) to use only the real part of matrix difference:

$$H_{22} = \left[\operatorname{Re}\left\{C - B^* \cdot A^{-1} \cdot B\right\}\right]^{-1}.$$

Unfortunately, the author has not known yet a strict substantiation of such approach. However, its neglection results in a situation, when obtained dispersion of estimations of non-energetic parameters of signals of M sources correspond to only cophase or to anti-phase situations of signals reception [3].

In case of a single source the problem of investigation of estimation accuracy is much simplified. Thus $A = a_1$, $Y = \begin{bmatrix} x_1 & z_1 \end{bmatrix}^T$,

$$P = S \blacksquare F = \begin{bmatrix} S_1(z_1) \cdot \begin{bmatrix} F_1(x_1) \\ \vdots \\ F_R(x_1) \end{bmatrix} \\ \hdashline \vdots \\ \hdashline S_T(z_1) \cdot \begin{bmatrix} F_1(x_1) \\ \vdots \\ F_R(x_1) \end{bmatrix} \end{bmatrix};$$

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$$\frac{\partial P}{\partial Y}=\left[\begin{array}{c:c} S_1(z_1)\cdot\begin{bmatrix}\dfrac{\partial F_1(x_1)}{\partial x_1}\\ \vdots \\ \dfrac{\partial F_R(x_1)}{\partial x_1}\end{bmatrix} & \dfrac{\partial S_1(z_1)}{\partial z_1}\cdot\begin{bmatrix}F_1(x_1)\\ \vdots \\ F_R(x_1)\end{bmatrix} \\ \hdashline \vdots & \vdots \\ \hdashline S_T(z_1)\cdot\begin{bmatrix}\dfrac{\partial F_1(x_1)}{\partial x_1}\\ \vdots \\ \dfrac{\partial F_R(x_1)}{\partial x_1}\end{bmatrix} & \dfrac{\partial S_T(z_1)}{\partial z_1}\cdot\begin{bmatrix}F_1(x_1)\\ \vdots \\ F_R(x_1)\end{bmatrix}\end{array}\right];$$

$$P^T\cdot P=\sum_{t=1}^{T}S_t^2(z_1)\cdot\left\{\sum_{r=1}^{R}F_r^2(x_1)\right\};$$

$$(A^*\otimes P^T)\cdot\frac{\partial P}{\partial Y}=$$

$$=a^*\cdot\left[\sum_{t=1}^{T}S_t^2(z_1)\cdot\left\{\sum_{r=1}^{R}F_r^2(x_1)\cdot\frac{\partial F_r(x_1)}{\partial x_1}\right\}\ \middle|\ \left\{\sum_{r=1}^{T}S_t^2(z_1)\cdot\frac{\partial S_t(z_1)}{\partial z_1}\right\}\left\{\sum_{r=1}^{R}F_r^2(x_1)\right\}\right];$$

$$\left(\frac{\partial P}{\partial Y}\right)^T\cdot(A\otimes P)=$$

$$=a\cdot\begin{bmatrix}\displaystyle\sum_{t=1}^{T}S_t^2(z_1)\cdot\left\{\sum_{r=1}^{R}F_r^2(x_1)\cdot\frac{\partial F_r(x_1)}{\partial x_1}\right\}\\ \displaystyle\left\{\sum_{r=1}^{T}S_t^2(z_1)\cdot\frac{\partial S_t(z_1)}{\partial z_1}\right\}\left\{\sum_{r=1}^{R}F_r^2(x_1)\right\}\end{bmatrix};$$

$$\left(\frac{\partial P}{\partial Y}\right)^T\cdot(AA^*\otimes 1_{RT})\cdot\frac{\partial P}{\partial Y}=$$

$$a^2\cdot\left[\begin{array}{c}\displaystyle\sum_{t=1}^{T}S_t^2(z_1)\cdot\left\{\sum_{r=1}^{R}\cdot\left(\frac{\partial F_r(x_1)}{\partial x_1}\right)^2\right\}\\ \displaystyle\left\{\sum_{r=1}^{T}S_t(z_1)\cdot\frac{\partial S_t(z_1)}{\partial z_1}\right\}\left\{\sum_{r=1}^{R}F_r(x_1)\cdot\left(\frac{\partial F_r(x_1)}{\partial x_1}\right)\right\}\end{array}\right.;$$

$$\left.\begin{array}{c}\displaystyle\left\{\sum_{r=1}^{T}S_t(z_1)\cdot\frac{\partial S_t(z_1)}{\partial z_1}\right\}\left\{\sum_{r=1}^{R}F_r(x_1)\cdot\left(\frac{\partial F_r(x_1)}{\partial x_1}\right)\right\}\\ \displaystyle\left\{\sum_{r=1}^{T}\left(\frac{\partial S_t(z_1)}{\partial z_1}\right)^2\right\}\left\{\sum_{r=1}^{R}F_r^2(x_1)\right\}\end{array}\right];$$

The analysis of resulted relations allows to make a conclusion about the possibility of increase of the accuracy of the solution of range and angle measurement at the expense of minimization of value $\left\{\sum_{r=1}^{T}S_t(z_1)\cdot\frac{\partial S_t(z_1)}{\partial z_1}\right\}\left\{\sum_{r=1}^{R}F_r(x_1)\cdot\left(\frac{\partial F_r(x_1)}{\partial x_1}\right)\right\}$ – for identical DAA channels or sum $\left\{\sum_{r=1}^{R}\sum_{t=1}^{T}S_t(z_1)\cdot\frac{\partial S_t(z_1)}{\partial z_1}\cdot F_r(x_1)\cdot\left(\frac{\partial F_r(x_1)}{\partial x_1}\right)\right\}$ - in more general case, when in each t – gate its own set of directivity characteristics of channels is formed and the description of the responses of gates in channels are non-identical.

REFERENCES

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