COURSES & LECTURES

New Matrix Operations for DSP

Dr. Vadim Slyusar

Module	Status
Lecture	

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DESCRIPTION
This lecture presents the basic concepts of new matrix operations and related applications for digital beamforming. This lecture can be used for radar system, smart antennas for wireless communications, and other systems applying digital beamforming. It's intended for individuals new to the field who wish to gain a basic understanding in this area. For additional information, check out the reference material presented at the end of this lecture.

PREREQUISITES
Matrix theory and digital beamforming.

INTENDED AUDIENCE
Individuals interested in digital signal processing.

ESTIMATED TIME
30 minutes

View the complete TechOnLine University Course Guide.

AUTHOR

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Dr.Vadim Slyusar was born in Poltava, Ukraine, on October 15, 1964. Dr.V.Slyusar has 16 years of research experience in the areas of radar systems, smart antennas for wireless communications and digital beamforming. He earned his Ph.D. in 1992, Dr.D. in 2000 and has 25 patents and 152 publications in these areas. He is chief of a research department in the R&D Group for Electromechanics and Pulsed Power (Kyiv, Ukraine), and is an authority in digital signal processing for radar applications. You can contact author at swadim@profit.net.ua

Keywords: university,

Matrix Technique for DSP Advantages

- The compactness of mathematical models of physical systems;
- The best presentation of essence of signal processing algorithms;
- Computer time economy.

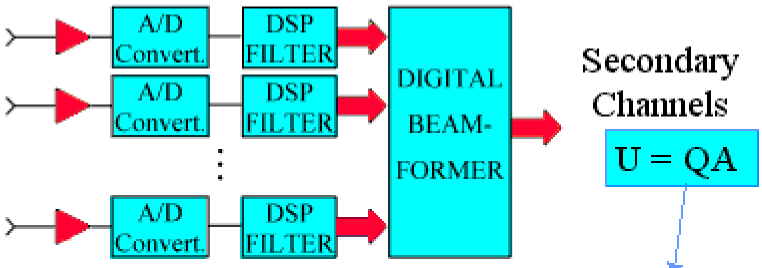
This matrix is especially advantageous for the digital multichannel systems of data processing!

Welcome to the TechOnline lecture about the theory of new matrix operations for digital signal processing.

The application of matrices, as you known, allows us to effeciently construct a model of a physical system and to formulate the essence of algorithms for processing signals. Matrix means is especially advantageous for solving the problems associated with the analysis, synthesis, capture, and data processing of complex multichannel systems.

This lecture concentrates on the application of matrices in radar systems with digital beamforming. However, these matrices also can be utilized for any system implementing digital beamforming. For example, in acoustics, hydroacoustics, cellular radio communication, ultrasonic medical diagnostics, radio astronomy, etc. In addition, these new matrix procedures can be useful for three-dimensional, image visualization systems.

Radar System with Digital Beamforming



$$U = \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \vdots \\ \dot{U}_R \end{bmatrix};\quad Q = \begin{bmatrix} Q_1(x_1) & Q_1(x_2) & \cdots & Q_1(x_M) \\ Q_2(x_1) & Q_2(x_2) & \cdots & Q_2(x_M) \\ \vdots & \vdots & \vdots & \vdots \\ Q_R(x_1) & Q_R(x_2) & \cdots & Q_R(x_M) \end{bmatrix};\quad A = \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \vdots \\ \dot{a}_M \end{bmatrix}$$

The simplest form of radar with digital beamforming is a one-coordinate N-elements array pattern with an A/D (analog-to-digital) converter, with DSP signal filtering in each channel, and beam sheaf forming with the help of fast Fourier transforms.

Voltage arrays resulting from beamforming with exposure to signals of M-sources in matrix form, ignoring noise, are written as U = FA, where U is a vector of complex digital beamformer response voltages. F is an R x M matrix of the directivity characteristics of R secondary channels at x coordinates of M sources. A is the vector of complex amplitudes of M source signals.

2 Coordinate Radar System

(1) $$U = Q \cdot \begin{bmatrix} \dot{a}_1 & 0 & \cdots & 0 \\ 0 & \dot{a}_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \dot{a}_M \end{bmatrix} \cdot F^T, \qquad F = \begin{bmatrix} F_1(\omega_1) & F_1(\omega_2) & \cdots & F_1(\omega_M) \\ F_2(\omega_1) & F_2(\omega_2) & \cdots & F_2(\omega_M) \\ \vdots & \vdots & \cdots & \vdots \\ F_T(\omega_1) & F_T(\omega_2) & \cdots & F_T(\omega_M) \end{bmatrix};$$

(2) $$U = \left[\begin{array}{ccc:c:ccc} Q_1(x_1) & \cdots & Q_1(x_M) & \cdots & & & \\ \vdots & \cdots & \vdots & & & 0 & \\ Q_R(x_1) & \cdots & Q_R(x_M) & & & & \\ \hdashline & \vdots & & \cdots & & \vdots & \\ \hdashline & & & & Q_1(x_1) & \cdots & Q_1(x_M) \\ & 0 & & \cdots & \vdots & \cdots & \vdots \\ & & & & Q_R(x_1) & \cdots & Q_R(x_M) \end{array}\right] \left[\begin{array}{cccc} F_1(\omega_1) & 0 & \cdots & 0 \\ 0 & F_1(\omega_2) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & F_1(\omega_M) \\ \hdashline & \vdots & & \\ \hdashline F_T(\omega_1) & 0 & \cdots & 0 \\ 0 & F_T(\omega_2) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & F_T(\omega_M) \end{array}\right] \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \vdots \\ \dot{a}_M \end{bmatrix}$$

Traditional matrix techniques, used with digital beamforming two-coordinate radars, make it possible to compact the expressions shown here in (1), which has no unity with the one coordinate model U = PA we’ve just discussed. The retention of one-channel variant structure leads to unwieldy expressions or to the introduction of new matrix operations, such as rows diagonalization or columns diagonalization of the matrices, as we see here in (2).

When the measuring coordinates are increased, the amount of defects indicated becomes more evident, and this is a serious problem.

In order to resolve this problem, we offer new operations of matrix multiplication. This lecture is devoted to their consideration.

Slyusar Product Matrix

The Example

$$A \square B = \left[a_{ij} \cdot B_i\right]$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$A \square B = \left[\begin{array}{ccc:ccc} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} & a_{12}b_{11} & a_{12}b_{12} & a_{12}b_{13} \\ a_{21}b_{21} & a_{21}b_{22} & a_{21}b_{23} & a_{22}b_{21} & a_{22}b_{22} & a_{22}b_{23} \\ a_{31}b_{31} & a_{31}b_{32} & a_{31}b_{33} & a_{32}b_{31} & a_{32}b_{32} & a_{32}b_{33} \end{array}\right]$$

When defining two matrices with an identical amount of lines, the Slyusar product is a matrix obtained by multipling each element in the left matrix by a row in the right matrix, which corresponds to a number in the left matrix.

A symmetrical alternative for the Slyusar product is a transposed Slyusar product. We'll consider this next.

Transposed Slyusar Product Matrix

The Example

$$A \blacksquare B = \left[a_{ij} \cdot B_{j}\right]$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$A \blacksquare B = \begin{bmatrix} a_{11}\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} & a_{12}\begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix} & a_{13}\begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} \\ \cdots & \cdots & \cdots \\ a_{21}\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} & a_{22}\begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix} & a_{23}\begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} \end{bmatrix}$$

When defining two matrices with an identical number of columns, the transposed Slyusar product is a matrix obtained by multipling each element in the left matrix, with a column in the right matrix, which corresponds to a number in the left matrix.

Proposed Matrix Operations

Fundamental Properties

Theorem 1 $(A \square B)(C \blacksquare D) = (A\,C) \circ (B\,D)$

"$\circ$" denotes element-wise multiplication(Hadamard product)

Theorem 2 $(A \otimes B)(C \blacksquare D) = (A\,C) \blacksquare (B\,D)$

"$\otimes$" denotes Kronecker product

Theorem 3 $(A \square B)(C \otimes D) = (A\,C) \square (B\,D)$

Theorem 4 $(A \square B)^T = A^T \blacksquare B^T$

Theorem 5
$$(A \square L)(B \otimes M)\ldots(C \otimes S)(K \blacksquare T) = \\ = (AB \;\ldots\; CK) \circ (LM \;\ldots\; ST)$$

The fundamental properties of these proposed matrix operations are presented here. We'll discuss each of them in more detail next.

Proposed Matrix Operations

Fundamental Properties

$$\begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & a_p \end{bmatrix} \cdot B = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} \square B$$

$$a \blacksquare b = a \otimes b$$

$$a^T \square b^T = a^T \otimes b^T$$

$$a \square b = a \circ b$$

a and b is k-vector

$$\mathrm{invec}_M \left[(Q \blacksquare F) b \right] = F \left(b \square Q^T \right) = \left(b^T \blacksquare F \right) Q^T$$

$\mathrm{invec}_M [\bullet]$ **denote the inversion vectorization operator (new)**

The Example

$$\mathrm{invec}_3(A) = \begin{bmatrix} a_1 & a_4 & \cdots & a_{p-5} & a_{p-2} \\ a_2 & a_5 & \cdots & a_{p-4} & a_{p-1} \\ a_3 & a_6 & \cdots & a_{p-3} & a_p \end{bmatrix},$$

where

$$A = \left[a_1 \; a_2 \; a_3 \; a_4 \; a_5 \; a_6 \; \cdots \; a_{p-5} \; a_{p-4} \; a_{p-3} \; a_{p-2} \; a_{p-1} \; a_p \right]^T$$

Using the Slyusar product allows us to reduce the amount of computing operations required in widespread problems, when multipling diagonal matrix A by matrix B, which conforms with it through p X g rows.

For all of this, the transition to the Slyusar product results in a reduction of p times the multiplication operations, and allows us to completely exclude the pg(p-1) operation in the summation.

Four Coordinate Radar System

$$U = (Q \blacksquare V \blacksquare F \blacksquare S)A = \left[\dot{U}_{rntd} = \sum_{m=1}^{M} \dot{a}_m Q_r(x_m) V_n(y_m) F_t(\omega_m) S_d(z_m)\right] =$$

$$= \left(\begin{bmatrix} Q_1(x_1) & Q_1(x_2) & \cdots & Q_1(x_M) \\ Q_2(x_1) & Q_2(x_2) & \cdots & Q_2(x_M) \\ \vdots & \vdots & \cdots & \vdots \\ Q_R(x_1) & Q_R(x_2) & \cdots & Q_R(x_M) \end{bmatrix} \blacksquare \begin{bmatrix} V_1(y_1) & V_1(y_2) & \cdots & V_1(y_M) \\ V_2(y_1) & V_2(y_2) & \cdots & V_2(y_M) \\ \vdots & \vdots & \cdots & \vdots \\ V_N(y_1) & V_N(y_2) & \cdots & V_N(y_M) \end{bmatrix} \blacksquare \right.$$

$$\left. \blacksquare \begin{bmatrix} F_1(\omega_1) & F_1(\omega_2) & \cdots & F_1(\omega_M) \\ F_2(\omega_1) & F_2(\omega_2) & \cdots & F_2(\omega_M) \\ \vdots & \vdots & \cdots & \vdots \\ F_T(\omega_1) & F_T(\omega_2) & \cdots & F_T(\omega_M) \end{bmatrix} \blacksquare \begin{bmatrix} S_1(z_1) & S_1(z_2) & \cdots & S_1(z_M) \\ S_2(z_1) & S_2(z_2) & \cdots & S_2(z_M) \\ \vdots & \vdots & \cdots & \vdots \\ S_D(z_1) & S_D(z_2) & \cdots & S_D(z_M) \end{bmatrix} \right) \cdot \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \vdots \\ \dot{a}_M \end{bmatrix},$$

Where x_m, y_m, w_m, and z_m is angular coordinates, frequencies and ranges of signals sources respectively.

As you can see here, the matrix model of a four-coordinate radar, which measures the angular coordinates, velocity, and range of M sources, is obtained with the help of the transposed Slyusar product.

Comparing this model with a matrix model, based on a standard matrix product, this new model allows us to receive a considerable reduction in the amount of computing operations. This is very important, for example, for modeling such systems in the MatLab or MathCad packages.

Maximum Likelihood Method

$$\tilde{L} = \mathrm{tr}(U - PA)^{*}(U - PA) = \min,$$

$P = Q \blacksquare V \blacksquare F \blacksquare S$, tr denote the matrix trace operation

The measuring procedure in four - coordinate variant is reduced to maximization of expression

$$L = \mathrm{tr}\left(GUU^{*}\right), G = P\left(P^{T}P\right)^{-1}P^{T}$$

$$(Q \blacksquare V \blacksquare F \blacksquare S)^{T}(Q \blacksquare V \blacksquare F \blacksquare S) = (Q^{T}Q)\circ(V^{T}V)\circ(F^{T}F)\circ(S^{T}S)$$

$$G = P\left[(Q^{T}Q)\circ(V^{T}V)\circ(F^{T}F)\circ(S^{T}S)\right]^{-1}P^{T}$$

This new matrix operation significantly decreases computer time by using multisignal measurement methods in matrix form. Here, we see a well-know method, maximization likelihood, for reducing the maxmization of function L.

Calculating the quadratic form by identity is reduced to the Hadamard product. As a result, for a 32x32 antenna array in each channel with 32 synthesized frequency filters in 32 distance intervals, we can decrease the amount of multiplication by 8,004 times and the amount of summations by 8,456 times with respect to the initial notation. In this case, the number of multiplication operations are decreased by more than 268.845 billion, as compared to a four-coordinate model based on a traditional matrix product.

Next, we'll consider the Cramer-Rao bound.

The Cramer-Rao Bound

$$I = \frac{1}{\sigma^2} \cdot \left[\begin{array}{c|c} P^T \cdot P & (A^* \otimes P^T) \cdot \frac{\partial P}{\partial Y} \\ \hline \left(\frac{\partial P}{\partial Y}\right)^T \cdot (A \otimes P) & \left(\frac{\partial P}{\partial Y}\right)^T \cdot (AA^* \otimes 1_{RNTD}) \cdot \frac{\partial P}{\partial Y} \end{array} \right]$$

where σ^2 is the noise dispersion, $\frac{\partial P}{\partial Y} = \frac{\partial}{\partial vec(Y)} \otimes vec(P)$ —

— the *Neudecker derivative* of matrix P (P is the function of matrix Y),
For a 4 - coordinate radar system with digital beamforming

$P = Q \bullet V \bullet F \bullet S$ and 1_{RNTD} is identity RxNxTxD- matrix,

vec(•) - denote verctorization (stacking columns of a matrix to for a vector).

To evaluate the potential accuracy of a maximum likelihood algorithm measurement, the lower Cramer-Rao bound can be used. This is obtained by reversing the information in a Fisher matrix. The Fisher matrix for a four-coordinate radar is illustrated here, as well as for the one-coordinate problem. The advantage of these proposed matrix operations is that the one-coordinate case, by the simple substitution, can also be used on a multi-coordinate case.

Neudecker Derivative Matrix

For example: 3 coordinates and M signals sources

$$\frac{\partial P}{\partial Y} = \frac{\partial (Q \blacksquare V \blacksquare F)}{\partial Y} = \partial Q_y \circledast \partial V_y \circledast \partial F_y = \begin{bmatrix} Q_1 & Q_1 & \frac{\partial Q_1}{\partial q_1} \\ \vdots & \vdots & \vdots \\ Q_M & Q_M & \frac{\partial Q_M}{\partial q_M} \end{bmatrix} \circledast \begin{bmatrix} V_1 & \frac{\partial V_1}{\partial y_1} & V_1 \\ \vdots & \vdots & \vdots \\ V_M & \frac{\partial V_M}{\partial y_M} & V_M \end{bmatrix} \circledast \begin{bmatrix} \frac{\partial F_1}{\partial \omega_1} & F_1 & F_1 \\ \vdots & \vdots & \vdots \\ \frac{\partial F_M}{\partial \omega_M} & F_M & F_M \end{bmatrix},$$

$$\frac{\partial F_1}{\partial \omega_1} = \begin{bmatrix} \partial F_1(\omega_1)/\partial \omega_1 & \cdots \\ \vdots & \vdots \\ \partial F_T(\omega_1)/\partial \omega_1 & \cdots \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0 & F_1(\omega_2) & \cdots \\ \vdots & \vdots & \vdots \\ 0 & F_T(\omega_2) & \cdots \end{bmatrix}, \quad Y = [\omega_1 \ \ldots \ \omega_M \mid y_1 \ \ldots \ y_M \mid x_1 \ \ldots \ x_M]^T$$

Here's a Neudecker derivative matrix, in expression form, for the Fischer information matrix. It's remarkable that in the case of the Slyusar product, or its transposed variant, that the result of the Neudecker derivative can be factorized as shown here. As you can see, we've used the symbol of the modular transposed Slyusar product. Their definitions and properties are considered next.

Modular Slyusar Product Concepts

The Example

$$A \circledcirc B = [A_{ij} \square B_{ij}]$$

$$A \circledcirc B = \left[\begin{array}{c|c} A_{11} \square B_{11} & A_{12} \square B_{12} \\ \hline A_{21} \square B_{21} & A_{22} \square B_{22} \end{array}\right], \quad A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \left[\begin{array}{cc|cc} a_{1111} & a_{1211} & a_{1112} & a_{1212} \\ a_{2111} & a_{2211} & a_{2112} & a_{2212} \\ a_{3111} & a_{3211} & a_{3112} & a_{3212} \\ \hline a_{1121} & a_{1221} & a_{1122} & a_{1222} \\ a_{2121} & a_{2221} & a_{2122} & a_{2222} \\ a_{3121} & a_{3221} & a_{3122} & a_{3222} \end{array}\right]$$

$$B = \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right] = \left[\begin{array}{ccc|ccc} b_{1111} & b_{1211} & b_{1311} & b_{1112} & b_{1212} & b_{1312} \\ b_{2111} & b_{2211} & b_{2311} & b_{2112} & b_{2212} & b_{2312} \\ b_{3111} & b_{3211} & b_{3311} & b_{3112} & b_{3212} & b_{3312} \\ \hline b_{1121} & b_{1221} & b_{1321} & b_{1122} & b_{1222} & b_{1322} \\ b_{2121} & b_{2221} & b_{2321} & b_{2122} & b_{2222} & b_{2322} \\ b_{3121} & b_{3221} & b_{3321} & b_{3122} & b_{3222} & b_{3322} \end{array}\right]$$

Here, we see an illustration of these modular Slyusar product concepts.

Modular Transposed Slyusar Product

The Example $A \odot B = [A_{ij} \blacksquare B_{ij}]$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \odot \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix} = \begin{bmatrix} A_{11} \blacksquare B_{11} & A_{12} \blacksquare B_{12} \\ A_{21} \blacksquare B_{21} & A_{22} \blacksquare B_{22} \\ A_{31} \blacksquare B_{31} & A_{32} \blacksquare B_{32} \end{bmatrix}$$

Theorem 6 $[A \odot B]^T = A^T \boxdot B^T$

Here, you can see the essence of the modular transposed Slyusar product.

The modular variants of Slyusar products have specific properties which we'll demonstrate next.

Basic Properties

Theorem 7

$$\left(\left[A_{ji}\right] \textcircled{$\square$} \left[B_{ji}\right]\right)\left(\left[K_{ik}\right] \textcircled{$\blacksquare$} \left[M_{ik}\right]\right) = \left[P_{jk} = \sum_{i} \left\{\left(A_{ji} \cdot K_{ik}\right) \circ \left(B_{ji} \cdot M_{ik}\right)\right\}\right]$$

Theorem 8

$$\left(\left[A_{ji}\right] \textcircled{$\square$} \left[B_{ji}\right]\right) \cdot \left[K_{ik} \otimes M_{ik}\right] = \left[P_{jk} = \sum_{i} \left\{\left(A_{ji} \cdot K_{ik}\right) \square \left(B_{ji} \cdot M_{ik}\right)\right\}\right]$$

Theorem 9

$$\left[A_{ji} \otimes B_{ji}\right] \cdot \left(\left[K_{ik}\right] \textcircled{$\blacksquare$} \left[M_{ik}\right]\right) = \left[P_{jk} = \sum_{i} \left\{\left(A_{ji} \cdot K_{ik}\right) \blacksquare \left(B_{ji} \cdot M_{ik}\right)\right\}\right].$$

Here, we see a list of the basic properties of modular and modular transposed Slyusar products.

With the help of the Slyusar product procedure, a mathematical model of a multisectional array with digital beamforming can be formalized.

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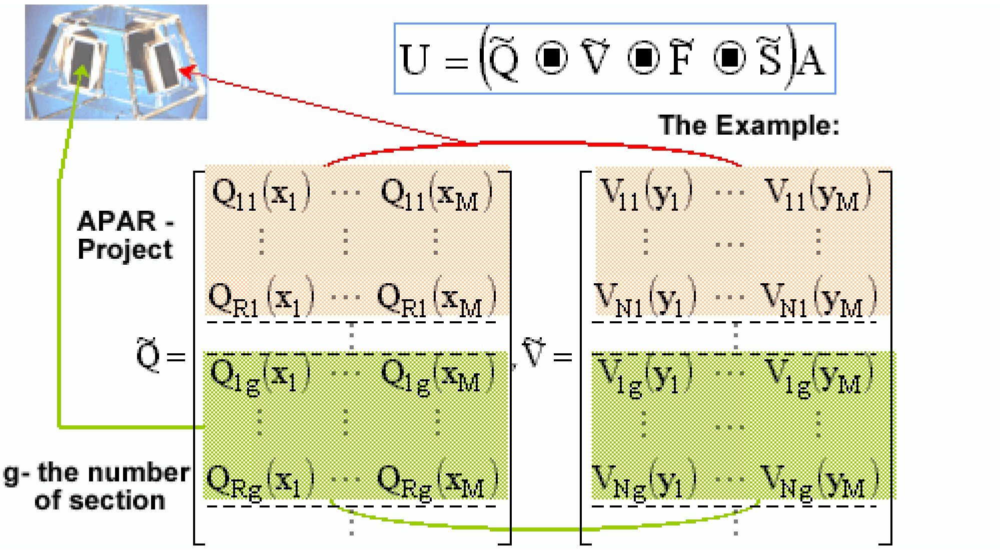
Mathematical Model

Multisectional Array



$$U = \left(\tilde{Q} \bullet \tilde{V} \bullet \tilde{F} \bullet \tilde{S}\right)A$$

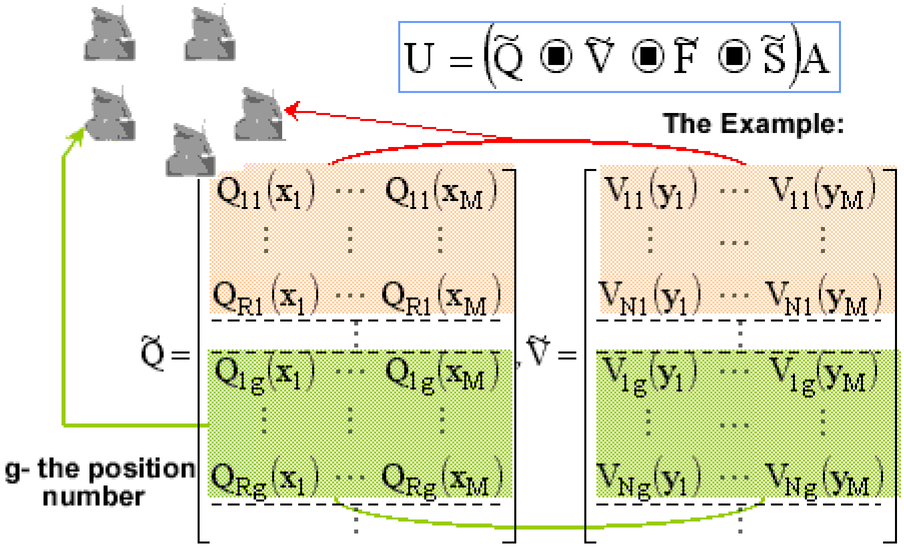
$$\tilde{Q} = \begin{bmatrix} Q_{11}(x_1) & \cdots & Q_{11}(x_M) \\ \vdots & \vdots & \vdots \\ Q_{R1}(x_1) & \cdots & Q_{R1}(x_M) \\ & \vdots & \\ Q_{1g}(x_1) & \cdots & Q_{1g}(x_M) \\ \vdots & \vdots & \vdots \\ Q_{Rg}(x_1) & \cdots & Q_{Rg}(x_M) \\ & \vdots & \end{bmatrix}, \quad \tilde{V} = \begin{bmatrix} V_{11}(y_1) & \cdots & V_{11}(y_M) \\ \vdots & \cdots & \vdots \\ V_{N1}(y_1) & \cdots & V_{N1}(y_M) \\ & \vdots & \\ V_{1g}(y_1) & \cdots & V_{1g}(y_M) \\ \vdots & \cdots & \vdots \\ V_{Ng}(y_1) & \cdots & V_{Ng}(y_M) \\ & \vdots & \end{bmatrix}$$

The mathematical model of a multisectional array with digital beamforming is illustrated here. Each section of the array has its own block of directivity characteristics, frequency, and range characteristics.

Mathematical Model

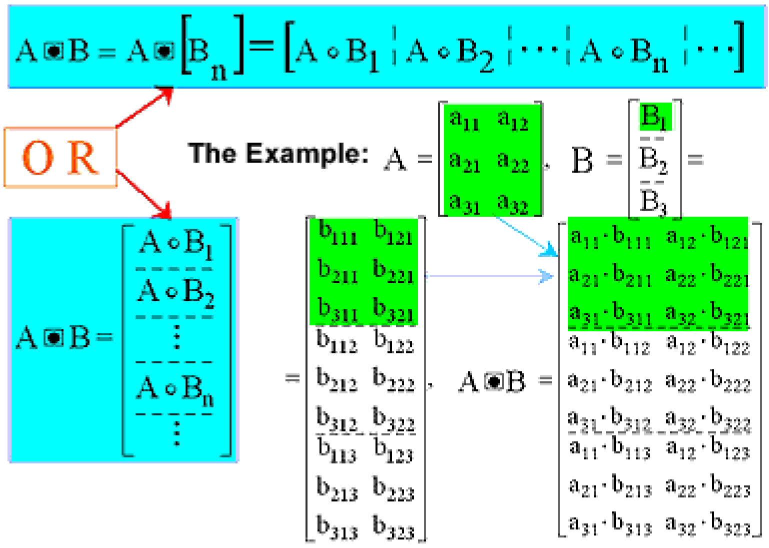
Multistatic Radar System

$$U = (\tilde{Q} \odot \tilde{V} \odot \tilde{F} \odot \tilde{S})A$$



Here, we see the mathematical model of a multistatic radar system with a digital beamforming antenna array. In this case, each position of the radar has its own block of directivity antenna array characteristics, frequency characteristics, and pulse signal shapes.

Penetrating Slyusar Product



These radar models are predicted based on the availability of the antenna pattern factorization and identity characteristics of the receiving channels. When such assumptions are impossible, the response formalization problem of a flat array can be carried out on the basis of a new penetrating Slyusar product. It can be determined for any matrix A and modular matrix B, where the dimensions of the modular matrix are the same as the dimensions of matrix A.

With the help of a penetrating Slyusar product, it’s possible to record the response of a three-coordinate radar to a single signal, taking into account that each receiving channel corresponds to its unique amplitude-frequency characteristic. This mathematical model is illustrated next.

Three Coordinates Radar System

Digital Beamforming for Nonidentical Channels (1 source)

$$U = \dot{a} \cdot (Q \boxdot F) = \dot{a} \cdot \left[Q \circ F_1 \,\vdots\, Q \circ F_2 \,\vdots\, \cdots \,\vdots\, Q \circ F_t \,\vdots\, \cdots \right]$$

$$Q = \begin{bmatrix} \dot{Q}_{11}(x,y) & \dot{Q}_{12}(x,y) & \cdots & \dot{Q}_{1R}(x,y) \\ \dot{Q}_{21}(x,y) & \dot{Q}_{22}(x,y) & \cdots & \dot{Q}_{2R}(x,y) \\ \vdots & \vdots & \vdots & \vdots \\ \dot{Q}_{R1}(x,y) & \dot{Q}_{R2}(x,y) & \cdots & \dot{Q}_{RR}(x,y) \end{bmatrix}$$

- matrix of the directivity characteristics of primary channels in azimuth and elevation angle planes (can not be factorized)

$$F = \left[\begin{matrix} \dot{F}_{111}(\omega) & \cdots & \dot{F}_{1R1}(\omega) \\ \vdots & \cdots & \vdots \\ \dot{F}_{R11}(\omega) & \cdots & \dot{F}_{RR1}(\omega) \end{matrix} \,\vdots\, \cdots \,\vdots\, \begin{matrix} \dot{F}_{11T}(\omega) & \cdots & \dot{F}_{1RT}(\omega) \\ \vdots & \cdots & \vdots \\ \dot{F}_{R1T}(\omega) & \cdots & \dot{F}_{RRT}(\omega) \end{matrix} \right]$$

The response of a three-coordinate flat digital antenna array of R x R elements can be stated by penetrating the face-splitting product of the matrices, without noise. Where U denotes a block-matrix of voltages of the response channels, A is a complex signal amplitude matrix, or vector for a single moment of time, Q is a matrix of the directivity characteristics of the primary channels in azimuth and elevation angle planes, which cannot be factorized, and F is a block-matrix of amplitude-frequency characteristics of T filters for R x R nonidentical reception channels.

For the selection of a single source on four coordinates, azimuth, elevation angle, frequency, and range, the response of a digital antenna array can be stated through a generalized Slyusar product or generalized transposed Slyusar product. We'll consider them next.

Generalized Slyusar Product

The Definition: $A \tilde{\square} B = \left[A_{ij} \boxdot \left[B_{i1} \quad B_{i2} \quad \cdots \quad B_{ig} \quad \cdots\right]\right]$

The Example:

$$A \tilde{\square} B = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1T} \\ A_{21} & A_{22} & \cdots & A_{2T} \\ \vdots & \vdots & \cdots & \vdots \\ A_{P1} & A_{P2} & \cdots & A_{PT} \end{bmatrix} \tilde{\square} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \cdots & \vdots \\ B_{P1} & B_{P2} & \cdots & B_{PG} \end{bmatrix} =$$

$$= \left[\begin{array}{c|c|c} A_{11} \boxdot \left[B_{11} \ \cdots \ B_{1G}\right] & \cdots & A_{1T} \boxdot \left[B_{11} \ \cdots \ B_{1G}\right] \\ A_{21} \boxdot \left[B_{21} \ \cdots \ B_{2G}\right] & \cdots & A_{2T} \boxdot \left[B_{21} \ \cdots \ B_{2G}\right] \\ \vdots & \vdots & \vdots \\ A_{P1} \boxdot \left[B_{P1} \ \cdots \ B_{PG}\right] & \cdots & A_{PT} \boxdot \left[B_{P1} \ \cdots \ B_{PG}\right] \end{array}\right].$$

As you can see, the concepts of the generalized Slyusar product are illustrated here. An alternative we should also consider is the generalized transposed face-splitting block-matrices product. We'll consider this next.

Generalized Transposed Slyusar Product

The Definition:

$$A \tilde{\blacksquare} B = \left[A_{ij} \boxdot \begin{bmatrix} B_{1j} \\ B_{2j} \\ \vdots \\ B_{Gj} \end{bmatrix} \right]$$

The Example:

$$A \tilde{\blacksquare} B = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1T} \\ A_{21} & A_{22} & \cdots & A_{2T} \\ \vdots & \vdots & \cdots & \vdots \\ A_{P1} & A_{P2} & \cdots & A_{PT} \end{bmatrix} \tilde{\blacksquare} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \cdots & \vdots \\ B_{P1} & B_{P2} & \cdots & B_{PG} \end{bmatrix} =$$

$$= \left[\begin{array}{c:c:c:c} A_{11} \boxdot \begin{bmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{P1} \end{bmatrix} & A_{12} \boxdot \begin{bmatrix} B_{12} \\ B_{22} \\ \vdots \\ B_{P2} \end{bmatrix} & \cdots & A_{1T} \boxdot \begin{bmatrix} B_{1G} \\ B_{2G} \\ \vdots \\ B_{PG} \end{bmatrix} \\ \hdashline \vdots & \vdots & \cdots & \vdots \\ \hdashline A_{P1} \boxdot \begin{bmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{P1} \end{bmatrix} & A_{P2} \boxdot \begin{bmatrix} B_{12} \\ B_{22} \\ \vdots \\ B_{P2} \end{bmatrix} & \cdots & A_{PT} \boxdot \begin{bmatrix} B_{1G} \\ B_{2G} \\ \vdots \\ B_{PG} \end{bmatrix} \end{array} \right]$$

This is the definition of a generalized transposed Slyusar product and an example of its application.

Four Coordinate Radar System

Digital Beamforming for Nonidentical Channels (1 source)

$$U = \left(Q \boxdot \left\{S \,\tilde{\square}\, F\right\}\right)\cdot \dot{a} = Q \boxdot \left[\, S_1 \boxdot F \,\vdots\, S_2 \boxdot F \,\vdots\, \cdots \,\vdots\, S_D \boxdot F \,\right]\cdot \dot{a}$$

$$Q = \begin{bmatrix} \dot{Q}_{11}(x,y) & \dot{Q}_{12}(x,y) & \cdots & \dot{Q}_{1R}(x,y) \\ \dot{Q}_{21}(x,y) & \dot{Q}_{22}(x,y) & \cdots & \dot{Q}_{2R}(x,y) \\ \vdots & \vdots & \vdots & \vdots \\ \dot{Q}_{R1}(x,y) & \dot{Q}_{R2}(x,y) & \cdots & \dot{Q}_{RR}(x,y) \end{bmatrix}$$

- matrix of the directivity characteristics of primary channels in azimuth and elevation angle planes (can not be factorized)

$$S = \left[\begin{array}{ccc|c|ccc} S_{111}(z) & \cdots & S_{1R1}(z) & & S_{11D}(z) & \cdots & S_{1RD}(z) \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ S_{R11}(z) & \cdots & S_{RR1}(z) & & S_{R1D}(z) & \cdots & S_{RRD}(z) \end{array}\right]$$

The response of a 4-coordinate flat digital antenna array of R x R elements can be stated through a penetrating face-splitting product of matrices, without noise. Where U denotes a block-matrix of response channel voltages, A is a complex signal amplitude matrix, or vector for a single moment in time, Q is a matrix of the directivity characteristics of primary channels in azimuth and elevation angle planes that cannot be factorized, and F is a block-matrix of amplitude-frequency characteristics of T filters for R x R nonidentical reception channels, and S is a block-matrix of range characteristics of D range gates.

Multistatic Radar System

Digital Beamforming for Nonidentical Channels (1 source)

$$U = \left(\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_P \end{bmatrix} \circ \begin{bmatrix} S_{11} & \cdots & S_{D1} \\ \vdots & \vdots & \vdots \\ S_{1P} & \cdots & S_{DP} \end{bmatrix} \circ \begin{bmatrix} F_{11} & \cdots & F_{T1} \\ \vdots & \vdots & \vdots \\ F_{1P} & \cdots & F_{TP} \end{bmatrix} \right) \cdot \dot{a}, \qquad U_{dtp} = \left(Q_p \circ S_{dp} \circ F_{tp} \right) \dot{a},$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_P \end{bmatrix} = \begin{bmatrix} \dot{Q}_{111}(x,y) & \cdots & \dot{Q}_{1R1}(x,y) \\ \dot{Q}_{211}(x,y) & \cdots & \dot{Q}_{2R1}(x,y) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R11}(x,y) & \cdots & \dot{Q}_{RR1}(x,y) \\ \hline \dot{Q}_{11P}(x,y) & \cdots & \dot{Q}_{1RP}(x,y) \\ \dot{Q}_{21P}(x,y) & \cdots & \dot{Q}_{2RP}(x,y) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R1P}(x,y) & \cdots & \dot{Q}_{RRP}(x,y) \end{bmatrix},$$

$$S_{dp} = \begin{bmatrix} S_{11dp}(z) & \cdots & S_{1Rdp}(z) \\ \vdots & \cdots & \vdots \\ S_{R1dp}(z) & \cdots & S_{RRdp}(z) \end{bmatrix},$$

$$F_{tp} = \begin{bmatrix} F_{11tp}(\omega) & \cdots & F_{1Rtp}(\omega) \\ \vdots & \cdots & \vdots \\ F_{R1tp}(\omega) & \cdots & F_{RRtp}(\omega) \end{bmatrix}.$$

Here, we see the mathematical model of a multistatic radar system with a digital beamforming antenna array for non-identical channels. In this case, each position of the radar has its own block of directivity antenna array characteristics, frequency characteristics, and pulse signal shapes.

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This concludes our lecture. Here are some references that were used in this lecture.

Congratulations, you have now completed

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January 26, 2000

Dear Vadim Slyusar:

This document is intended to verify that Vadim Slyusar has submitted the lecture, titled *"New matrix operations for DSP"* to TechOnLine Inc. TechOnLine has added, on the 9th of November 1999, the online version of this lecture to its TechOnLine University web site as an educational lecture.

On behalf of TechOnLine I want to thank you, Vadim, for all your time and efforts on this project.

Sincerely,

Mike Strange

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