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THE DISCRETE HILBERT FILTRATION OF PULSE SIGNALS

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According to current concepts, the complex-valued representation of physical signals simplifies the procedures of estimating their parameters. The modern computer facilities make it possible to carry out in real time even such an efficient means of transformation of real signals into complex ones as the discrete Hilbert transformation (DHT). This problem has been discussed in literature, however, most attention was paid to DHT of continuous signals [1–4] while the corresponding pulse processing is still not fully understood.

It is well known that in the process of sliding DHT the source real-valued set of samples for the analog-to-digital signal conversion is regarded as one of the quadrature components while the other is created on its basis by the weighted summation [1,2]. In the situations when the signal is present during the whole DHT "window", the artificially created quadrature corresponds to the ideal one with the accuracy defined by the order of the Hilbert digital filter. However, in the case of pulse signals it may happen sooner or later that the DHT window covers the signal sample only in part, and as a result the generation of the complex sequence occurs with some errors. It is essential that in the produced complex-valued response the distortions at its leading and trailing edge are inherent in only one quadrature.

Thus, in rigorous studies (for instance, in the problem of precise measurements of delay time) we have to take into account the differences in analytical description of the pulse signal envelope in the quadrature components. For instance, for discrete samples of voltages of a complex ratio pulse with nonmodulated carrier is may be done in the form:

$$\dot{U}_s = \begin{cases} a\,K(s-s_1)\exp\{j\,[\omega\,\Delta\,t\,(s-s_1)+\psi\,]\} + \dot{n}_s \text{ at } s_{\text{ef}} < s < s_{\text{nte}}; \\ a\,K(s-s_1)\exp\{j\,[\omega\,\Delta\,t\,(s-s_1)+\psi\,]\} + \dot{n}_s + \\ +j\,a\,\Delta\,K\,(s-s_1)\,\sin[\omega\,\Delta\,(s-s_1)+\psi\,] \text{ at } s_1 \le s \le s_{\text{ef}} \text{ and } s_{\text{nte}} \le s \le s_{\text{e}}; \\ \dot{n}_s \text{ at } s < s_1 \text{ and } s > s_{\text{e}}, \end{cases} \tag{1}$$

where s is the number of a complex sample; a is the signal amplitude; s_1 is the number of the first of the ADC samples during the existence time of the radio pulse; s_{ef} is the number of the latest sample of the ADC obtained at its leading edge; s_{nte} is the number of the first sample of the ADC occuring at the signal trailing edge; s_{e} is the moment of the pulse termination; $K(s-s_1)$ is, referenced to its maximum value, the discrete envelope of the source real-valued signal; $\Delta K(s-s_1)$ is the DHT error at the leading and trailing edge of the pulse (distortion of the normalized discrete function of the envelope due to the DHT transient process in the "sin"th quadrature); ω is the "pulse-filling" frequency; Δt is the period of signal digitization; ψ is the signal initial phase; and $\dot{n}_s$ is the noise complex value.

It is obvious that for a known function $K(s-s_1)$ the correction $\Delta K(s-s_1)$ may be always calculated beforehand. An example of practical implementation of this approach is a multisignal version of the distance-measuring process which has been synthesized by us by analogy with [5] based on the maximum likelihood method. The gist is to maximize, in the conditions of the prosessing "window" sliding along the ACD sampling set, a functional of the form

$$F_{\text{M}} = -\,\text{D}/\text{D}_{\text{M}} = \max \tag{2}$$

where D_M is determinant of the matrix $[\dot{Q}_{ij}]$, $i, j = 1, ..., M$ while D is determinant of the matrix which is different from $[\dot{Q}_{ij}]$ by the first row and column with entries 0, $\dot{W}_1^*$, ..., $\dot{W}_M^*$ and 0, $\dot{W}_1$, ... $\dot{W}_M$, respectively. Here

$$\dot{W}_n = \sum_{s=s_n}^{s_n+N-1} (U_s^c + j\, U_s^s)\, \dot{K}_{sn}^*;\ \dot{Q}_{mn} = \dot{Q}_{mn}^* = \sum_{s=s_m}^{s_n+N-1} \dot{K}_{sm}\, \dot{K}_{sn}^* \quad ;$$

$$\dot{K}_{sm} = [K_{sm}^c + j\, K_{sm}^s]\, [\cos p_{sm} + j \sin p_{sm}] =$$

$$= K_{sm}^c \cos p_{sm} - K_{sm}^s \sin p_{sm} + j\ [K_{sm}^s \cos p_{sm} + K_{sm}^c \sin p_{sm}]$$

is the generalized complex envelope; * is the symbol of complex conjugate; U_s^c, U_s^s are quadrature components of the signal mixture voltages produced in the course of DHT of the ADC sth sample; N is pulse duration in the ADC samplings (assumed to be equal for all signals); the argument p_{sm} describes the variation of the mth carrier frequency and phase over time (1); while $K_{sm}^c = K^c(s - s_m)$, $K_{sm}^s = K^c(s - s_m) + \Delta K(s - s_m)$ represents the behavior of the mth pulse envelope in the quadrature components after DHT of the source signal.

It should be noted that there also exists a vast class of pulse signals whose transient processes are expressed in a less explicit form than in the above case. The smoothing of the transient processes is observed not only due to narrowing the envelope spectrum of the source signal but also in the case of increasing the order of the Hilbert filter due to expanding its passband. Still the role of the latter factor is less significant since the extension of the transmission band reduces in our case mainly to improvement of rectangularity of the filter AFR.

Thus, in a number of applications dealing with narrowband pulse signals, when the signal duration exceeds the Hilbert filter order, we may introduce an assumption about coincidence between the envelopes in the response quadratures and the source envelope. It will be even more justified if all subsequent processing is based on the model of signal whose duration is less than the actual one, so that transient deformations of the envelope emerging from the noise fluctuations remain outside. In the sampled distance measuring problems which may be reduced to solution of an M-order algebraic equation (where M is the number of sources), the necessary condition states that the signal duration must exceed the Hilbert filter order no less than by $M + 1$ samples with the proviso that in the measurement we use a filtered signal sample in the neighborhood of the envelope maximum.

And, eventually, another approach is possible when, instead of the envelope of the source real-valued signal, in subsequent processing we employ a function which is common for both quadratures and corresponds to the module of the complex-valued response of the DHT sliding "window" $\hat{K}_{sm} = \sqrt{K_{sm}^{c\,2} + K_{sm}^{s\,2}}$.

The error of such an approximation is quite predictable but its evaluating requires an additional study. This technique makes the procedures of signal parameter measurement much easier. For instance, instead of the generalized complex envelope $\dot{K}_{sm}$ and its complex conjugate $\dot{K}_{sm}^*$ in (2) the following variables are to be used:

$$\dot{K}_{sm} = \hat{K}_{sm}\, \{\cos p_{sm} + j \sin p_{sm}\},\ \dot{K}_{sm}^* = \hat{K}_{sm}\, \{\cos p_{sm} - j \sin p_{sm}\}.$$

As may be seen from the above, the suggested approaches guarantee an acceptable quality of DHT in the case of solitary pulses.

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