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VOLUME 40 NUMBER 11 1997

RADIOELECTRONICS AND COMMUNICATIONS SYSTEMS

Radioelectronics and Communications Systems
Vol. 40, No. 11, pp. 16-22, 1997

Izvestiya VUZ. Radioelektronika
Vol. 40, No. 11, pp. 32–42, 1997
UDC 621.396.967

RANGING AT ACCURACY PROPORTIONAL TO THE RADIO PULSE FILLING FREQUENCY AND LENGTH

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The paper has considered the methods of raising the accuracy of ranging proportional to the radio pulse frequency filling and length based on the formation in the receiver of a multi-frequency signal and its matched processing in the mode of the "sliding" window.

The possibility of acquiring data contained in their "thin" structure is a distinctive feature of assessing the parameters of signal sources in the case of analog-to-digital conversion (ADC) of radio pulses [1]. As applied to range finding its record is manifested in achievement of measurement accuracy proportional not to the spectrum width of the signal $\Delta\omega$ but its carrier ω under the conditions that $\omega \gg \Delta\omega$ [2]. However, the presence of the data on the signal initial phase is required [1, 2]. Otherwise, range finding at an accuracy proportional to its filling frequency until now could be done only with shifting. Procedures of optimal summing and other methods of shaping correlation radio functions whose maximum position depends on the radio pulse initial phase are discussed.

The objective of the present article is consideration of measuring procedures ensuring, even with the unknown initial phase of the signal, lack of bias of the delay time estimates with variance inversely proportional to the radio signal filling frequency and length.

Having assumed the complex representation of voltages we will take as known the initial phase and frequency of the radio pulse. Then the Cramer-Rao lower bound for the range finding error variance using the numerical method without account for the noise correlation distributed following the normal law and quantization-digitization errors can be expressed in the form:

$$\sigma^2_{b\,\Delta t} \geq \frac{\sigma_n^2}{a^2} \times \left\{ \sum_{s=b}^{b+N} [K'(s-b)]^2 - \frac{\left[\sum\limits_{s=b}^{b+N} K(s-b)\, K'(s-b)\right]^2}{\sum\limits_{s=b}^{b+N} K^2(s-b)} + \omega^2 \sum_{s=b}^{b+N} K^2(s-b) \right\}^{-1}, \tag{1}$$

where a is the signal amplitude, σ_n^2 variance of noises in the quadrature component of its voltages, $K(s-b)$ is the envelope discrete function of the radio pulse normalized to its maximum (assumed

$$K(0) = K(N) < \Delta U,$$

ΔU is the ADC quantization step), b is the estimate of the moment of signal initiation in ADC samplings,

$$K'(s-b) = \frac{\partial K(s-b)}{\partial b},$$

ω is the radio pulse filling frequency, N is its length in digitization periods Δt.

With the unknown initial phase the delay time of the complex radio pulse is fixed with the error variance similar to the one shown in [3]:

$$\sigma^2_{b\,\Delta t} \geq \frac{\sigma_n^2}{a^2}\left\{ \sum_{s=b}^{b+N} [K'(s-b)]^2 - \frac{\left[\sum_{s=b}^{b+N} K(s-b)\,K'(s-b)\right]^2}{\sum_{s=b}^{b+N} K^2(s-b)} \right\}^{-1}. \tag{2}$$

In this case the dependence of the range evaluation variance on frequency disappears although a number of elements of the Fisher initial information matrix generating (2) also contain ω as a free constant. One can be convinced having calculated partial derivatives of the information equivalent of the likelihood functional

$$F = \sum_{s=b}^{b+N} \left[\quad \{ U_s^c - a\,K(s-b)\cos(\omega\Delta t(s-b)+\phi) \}^2 + \right.$$

$$\left. + \{ U_s^s - a\,K(s-b)\sin(\omega\Delta t(s-b)+\phi) \}^2 \right] \tag{3}$$

using unknown $a^c = a\cos\phi$, $a^s = a\sin\phi$ (ϕ is the initial phase) and having determined their mean values. The Fisher sought for matrix will be written down in the form

$$I = \frac{1}{\sigma_n^2}\begin{bmatrix} \sum_{s=b}^{b+N} K^2(s-b) & 0 & X \\ 0 & \sum_{s=b}^{b+N} K^2(s-b) & Y \\ X & Y & A \end{bmatrix} \tag{4}$$

where

$$A = a^2\omega^2 \sum_{s=b}^{b+N} K^2(s-b) + a^2 \sum_{s=b}^{b+N} [K'(s-b)]^2,$$

$$X = \omega a^s \sum_{s=b}^{b+N} K^2(s-b) - a^c \sum_{s=b}^{b+N} K'(s-b)\,K(s-b),$$

$$Y = -\omega a^c \sum_{s=b}^{b+N} K^2(s-b) - a^s \sum_{s=b}^{b+N} K'(s-b)\,K(s-b).$$

Note that mutual compensation in (2) of elements containing free factors ω and ω^2 occurs during matrix inversion (4) during the process of computing its determinant. The presence in mean values of second derivatives (3) of the operation of additional summation, for instance, using a certain index r on which frequency ω will depend could prevent this phenomenon.

The respective Fisher function

$$\tilde{I} = \frac{1}{\sigma_n^2}\begin{bmatrix} \sum_{r=1}^{R}\sum_{s=b}^{b+N} K_r^2(s-b) & 0 & \tilde{X} \\ 0 & \sum_{r=1}^{R}\sum_{s=b}^{b+N} K_r^2(s-b) & \tilde{Y} \\ \tilde{X} & \tilde{Y} & \tilde{A} \end{bmatrix}, \tag{5}$$

where

$$\mathcal{A} = a^2 \sum_{r=1}^{R} \sum_{s=b}^{b+N} [K_r'(s-b)]^2 + a^2 \sum_{r=1}^{R} \omega_r^2 \sum_{s=b}^{b+N} K_r^2(s-b),$$

$$\mathcal{X} = a^s \sum_{r=1}^{R} \omega_r \sum_{s=b}^{b+N} K_r^2(s-b) - a^c \sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r(s-b)\, K_r'(s-b),$$

$$\mathcal{Y} = -a^s \sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r(s-b)\, K_r'(s-b) - a^c \sum_{r=1}^{R} \omega_r \sum_{s=b}^{b+N} K_r^2(s-b),$$

while the Cramer-Rao bound generating it:

$$\sigma_{b\,\Delta t}^2 \geq \frac{\sigma_n^2}{a^2} \times$$

$$\times \left\{ \sum_{r=1}^{R} \sum_{s=b}^{b+N} [K_r'(s-b)]^2 - \frac{\left[\sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r'(s-b)\, K_r(s-b) \right]^2}{\sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r^2(s-b)} + D \right\}^{-1}, \tag{6}$$

where

$$D = \sum_{r=1}^{R} \omega_r^2 \sum_{s=b}^{b+N} K_r^2(s-b) - \left[\sum_{r=1}^{R} \omega_r \sum_{s=b}^{b+N} K_r^2(s-b) \right]^2 \times \left[\sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r^2(s-b) \right]^{-1}.$$

As could be expected, denominator (6) retained the dependence on the frequency which is absent in (2).

Comparing tables (4) and (5) one can get an unknown equivalent of the likelihood functional:

$$F = \sum_{r=1}^{R} \sum_{s=b}^{b+N} \left[\{U_{sr}^c - a_r K_r(s-b) \cos[\omega_r \Delta t (s-b) + \phi_r]\}^2 + \right.$$

$$\left. + \{U_{sr}^c - a_r K_r(s-b) \sin[\omega_r \Delta t (s-b) + \phi_r]\}^2 \right] = \min, \tag{7}$$

where r is the frequency ordinal number, $K_r(s-b)$ is the discrete function of the pulse envelope at the rth frequency, U_{sr}^c, U_{sr}^s are quadrature components of the signal voltages of the rth frequency, ϕ_r is the deviation of its initial phase from the rated value (here indices r of the amplitude and initial phase are added to create greater generality).

Following [4] we modify (7) having excluded unknown amplitudes of the signals. To this end we determine their values minimizing quadrature error (7) and substitute such into cross products of the terms forming discrepancies. As a result we will obtain an identical condition

$$F_M = \left\{ \sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r(s-b) [U_{sr}^c \cos[\omega_r \Delta t (s-b) + \phi_r] + \right.$$

$$\left. + U_{sr}^s \sin[\omega_r \Delta t (s-b) + \phi_r]] \right\}^2 + \left\{ \sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r(s-b) \times \right.$$

$$\left.\times \left[U^s_{sr} \cos \left[\omega_r \Delta t (s-b) + \phi_r\right] - U^c_{sr} \sin \left[\omega_r \Delta t (s-b) + \phi_r\right]\right]\right\}^2 = \max \tag{8}$$

In the given case the normalizing multiplier

$$\sum_{r=1}^{R} \sum_{s=b}^{b+N} K_r^2 (s-b)$$

has been omitted since it does not influence the position of the function F_M maximum.

Thus in mathematical terms the problem of ranging has been reduced to maximization (8) by selecting the estimate b in the mode processing window sliding along the arrays of samplings U^c_{sr} and U^s_{sr}. As for technical aspects of realizing the given algorithm they are no less substantial. The point is that externally the synthesized procedure can be classified as one of the varieties of multi-frequency sounding. However, for attaining accuracy (6) it is necessary to observe conditions of complete predictability of all $R - 1$ complex amplitudes relative to the Rth accepted as the reference. In multi-frequency sounding this requirement is hardly feasible especially in the case of a movable signal source.

Therefore, a distinctive feature of the approach being proposed is the use of a single sounding pulse and formation of a multi-frequency package in the receiver itself using the signal received, which can be fulfilled by converting its carrier at R intermediate frequencies. Neglecting the presence of noises, their mutual correlation and instabilities of the parameters of mixing-amplification units we can assume that amplitudes and initial phases of radio pulses shaped in such a way will be strictly linked with the values of similar parameters of echo-signals. This circumstance will make it possible to express values a_r and ϕ_r as estimates of the complex amplitude of one of the frequencies.

Computational costs during the measurement process could be the lowest in the case of complete coincidence of the amplitudes and initial phases of radio pulses. If the analog portion of the reception channel does not secure the identity of the indicated parameters, then correction of complex voltages of the signals obtained after the analog-to-digital conversion can be used to eliminate their differences using the following pattern:

$$\tilde{U}^c_{sr} = U^c_{sr}\,\alpha^c_r - U^s_{sr}\,\alpha^s_r, \quad \tilde{U}^s_{sr} = U^s_{sr}\,\alpha^c_r + U^c_{sr}\,\alpha^s_r.$$

The values of the correction coefficients α^c_r and α^s_r can be computed similarly to [5] using S samplings of the voltages of the reference signal in the reference channel $U^{c\,(s)}_{s,\,\mathrm{st}}$:

$$\alpha^c_r = \frac{\sum_{s=1}^{S} \left\{U^c_{s,\,\mathrm{st}}\, U^c_{sr} + U^s_{s,\,\mathrm{st}}\, U^s_{sr}\right\}}{\sum_{s=1}^{S} \left(U^{c\,2}_{sr} + U^{s\,2}_{sr}\right)}, \quad \alpha^s_r = \frac{\sum_{s=1}^{S} \left\{U^s_{s,\,\mathrm{st}}\, U^c_{sr} - U^c_{s,\,\mathrm{st}}\, U^s_{sr}\right\}}{\sum_{s=1}^{S} \left(U^{c\,2}_{sr} + U^{s\,2}_{sr}\right)}.$$

Note that a transmitter pulse that penetrated the receiver to shape a test train can be utilized, while with the spreading of variances of noises in quadrature components U^c_{sr} and U^s_{sr} the correction procedure can be supplemented by normalizing quadratures, corresponding to the rth frequency, to variance of their noise.

Among the conditions facilitating hardware implementation of the approach proposed a coincidence of the envelopes of all R signals plays an important role. In this case the analytical estimate of potential accuracy of measurements is noticeably simplified. Let us use the given circumstance for its detailed analysis.

Having excluded in (7) the dependence of the law of changing the envelope on the index r and having expressed the values of all frequencies, for instance, through their maximum ω_0 ($\omega_r = \gamma_r\, \omega_0$, $0 \le \gamma_r \le 1$), we obtain:

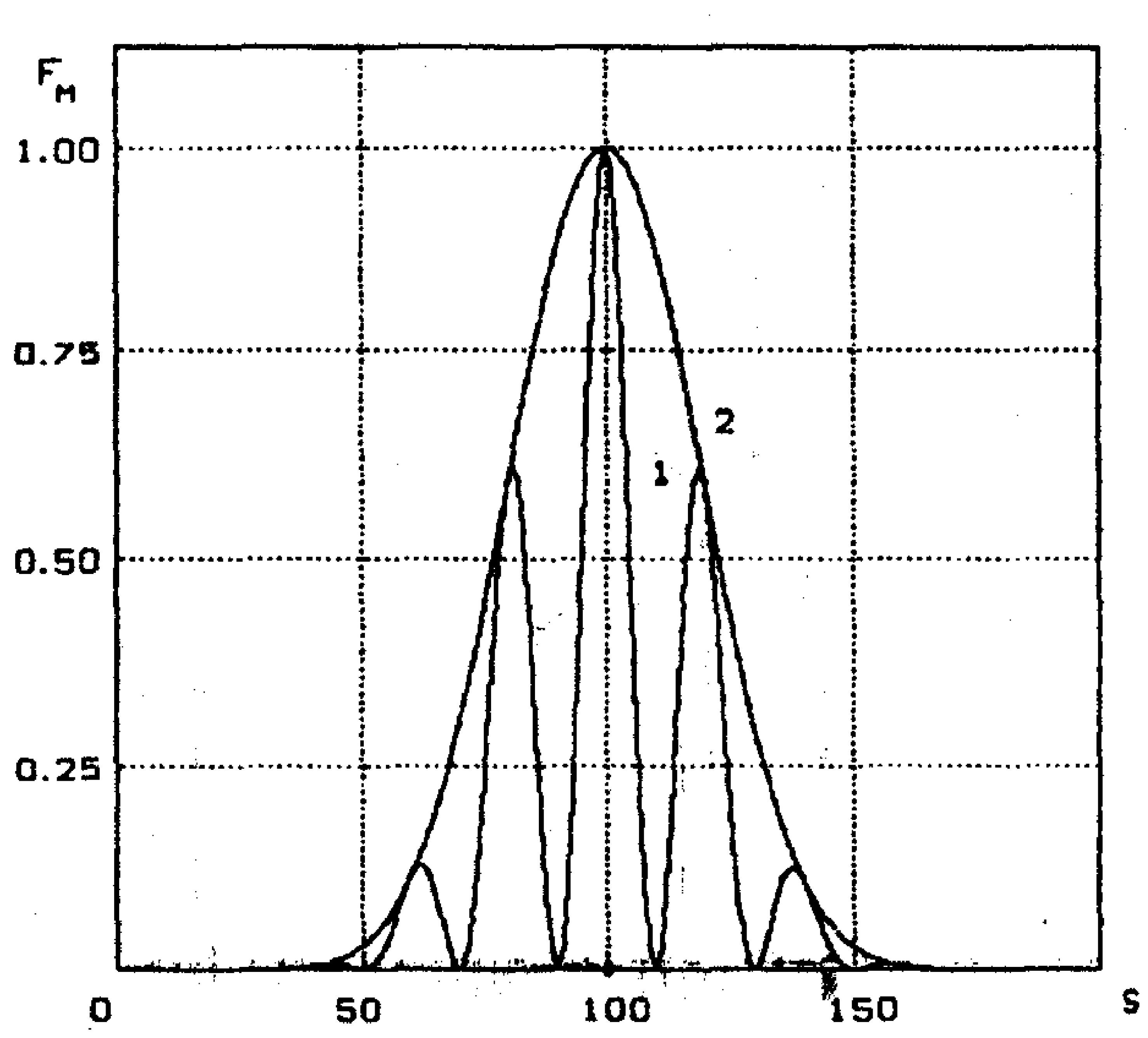

Fig. 1

$$\sigma^2_{b\Delta t} \geq \frac{\sigma_n^2}{a^2}\left\{\tilde{R}\left\{\sum_{s=b}^{b+N}[K'(s-b)]^2 - \frac{\left[\sum\limits_{s=b}^{b+N} K(s-b)\,K'(s-b)\right]^2}{\sum\limits_{s=b}^{b+N} K^2(s-b)}\right\} + \tilde{D}\,\omega_0^2\sum_{s=b}^{b+N} K^2(s-b)\right\}^{-1} \tag{9}$$

where

$$\tilde{D} = \sum_{r=1}^{R}\gamma_r^2 - \left[\sum_{r=1}^{R}\gamma_r\right]^2 R^{-1}.$$

For narrowband radio pulses with a symmetrical envelope it will be not difficult to achieve the excess of filling frequency ω_0 of the signal spectrum width; therefore the following inequality is observed:

$$a^2 R\sum_{s=b}^{b+N}[K'(s-b)]^2 \ll a^2\omega_0^2\sum_{s=b}^{b+N}K^2(s-b)\times\left[\sum_{r=1}^{R}\gamma_r^2 - \frac{\left[\sum\limits_{r=1}^{R}\gamma_r\right]^2}{R}\right] \tag{10}$$

and accuracy estimate (9), with the absence of anomalous spikes caused by side lobes of function F_M (8) will be similarly to (1) of the filling proportional frequency ω_0 and the length of radio pulses in digitization periods

$$\sigma^2_{b\Delta t} \approx \frac{\sigma_n^2}{a^2\omega_0^2}\left[\sum_{s=b}^{b+N}K^2(s-b)\left[\sum_{r=1}^{R}\gamma_r^2 - \frac{\left[\sum\limits_{r=1}^{R}\gamma_r\right]^2}{R}\right]\right]^{-1}. \tag{11}$$

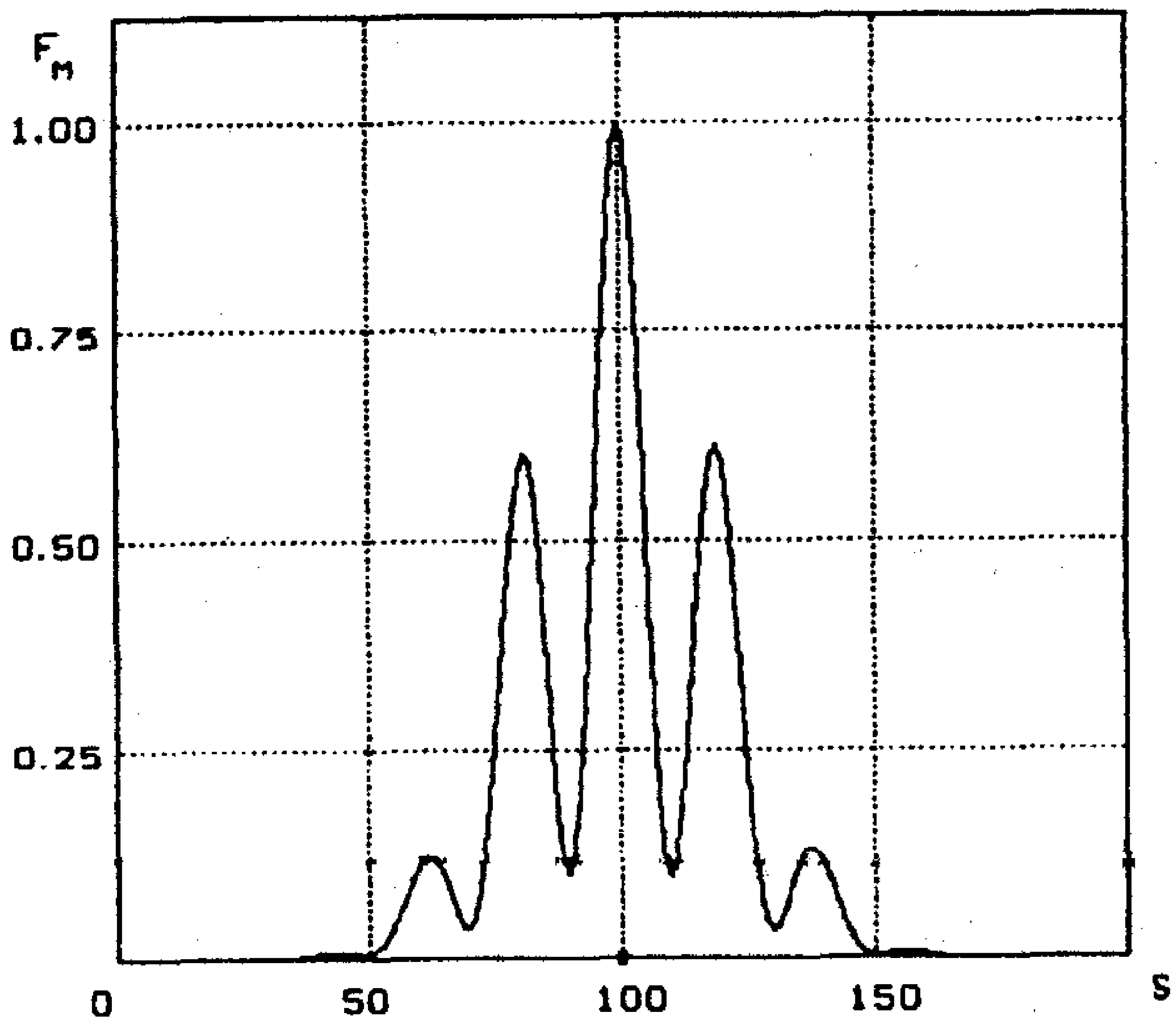

Fig. 2

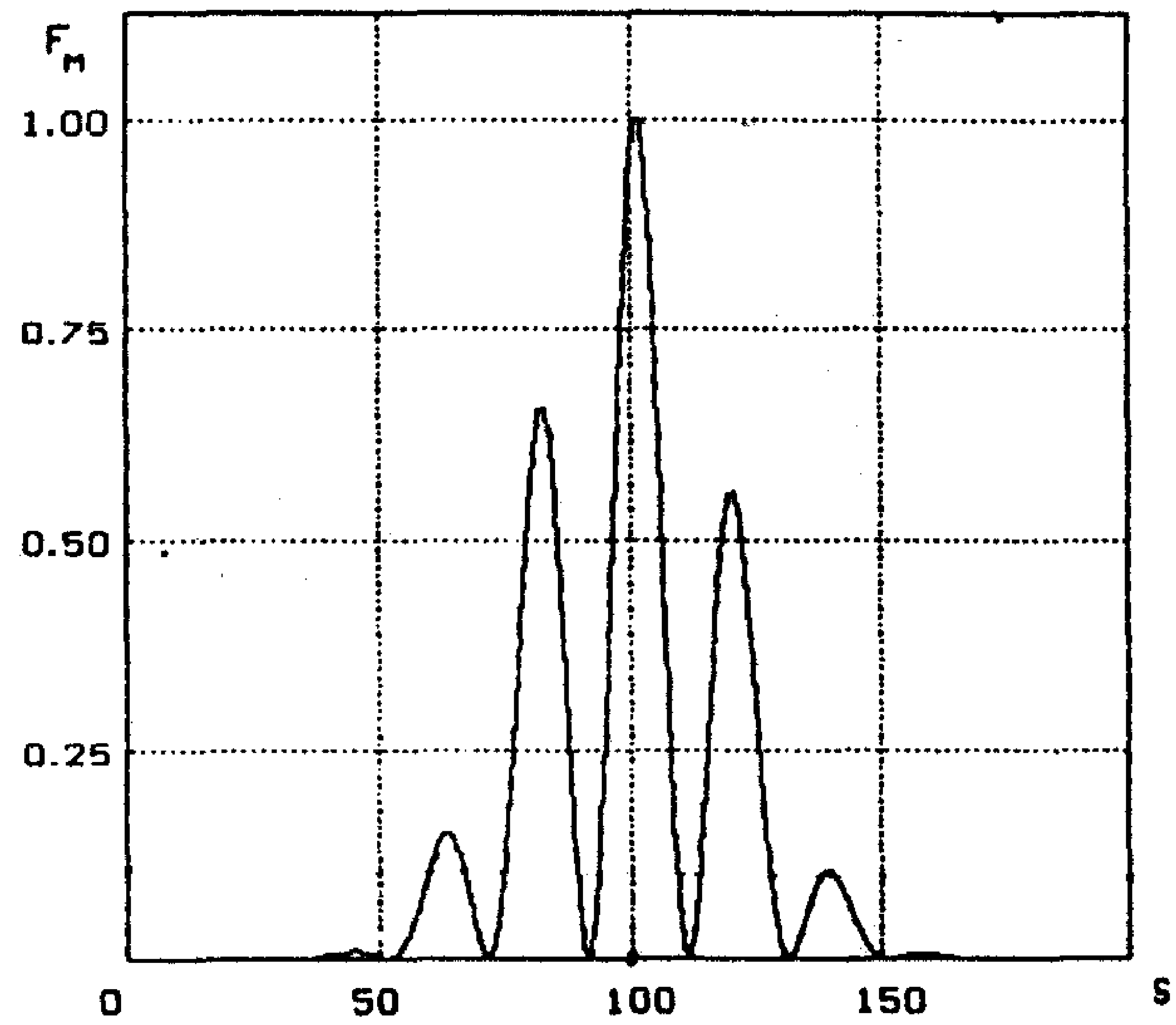

Fig. 3

To the end of confirming the validity of the results obtained we have conducted mathematical modeling of procedure (8) taking into account non-correlated Gaussian noises for the case of matched filtration of two-pulses. One should note that their quantity is most interesting for practical implementation since it requires minimal hardware costs.

Position *1* of Fig. 1 shows the result of shaping normalized function (8) under the condition that both signals started at the ADC 100th sampling, their amplitude was 100 quanta, the signal-to-noise ratio was 50, while other values incorporated into (8) assumed the following values

$$R = 2,\ K_r(s-b) = \sin^2 \frac{\pi}{N_r}(s-100),$$

$$r = 1;\ 2,\ N_1 = N_2 = 100,\ \phi_1 = \phi_2 = 0,\ \omega_1 \Delta t = 0.2\pi,\ \omega_2 \Delta t = 0.1\pi. \tag{12}$$

For comparison, position *2* shows the result of processing two radio pulses having coinciding filling frequencies. In this case, the width of the sliding window response is entirely determined by the envelope $\sin^2 \frac{\pi}{100}(s-100)$ and does not depend on the carrier.

As a result of statistical modeling we obtained confirmation of invariance $\sigma^2_{b\,\Delta t}$ to the signal initial phase, the presence of the dependence of ranging accuracy on the pulse length and frequency difference ω_1, ω_2. It is important, that with two-signal ranging, estimate accuracy can be identical with different combinations of the filling frequencies. A decisive factor in this case is not the value of frequencies but their difference $(\omega_1 - \omega_2)^2$.

Analysis of accuracy estimates (9) and (11) for two-signal processing reinforced by modeling results makes it possible to conclude that with the fixed maximum of frequency ω_0 the greatest measuring accuracy occurs in the case when one of the signals is a complex video pulse. The given conclusion completes the search for an answer to the question on how best to measure the signal delay time: either at the radio or video frequency. An option when both radio and video signals are used simultaneously is preferable. The corresponding measurement procedure is reduced to maximization by selecting the estimate b of the expression:

$$F_{\text{M}} = \left\{ \sum_{s=b}^{b+N} K(s-b)\left[U^c_{s_0} + U^c_{s_\omega} \cos \omega \Delta t (s-b) + U^s_{s_\omega} \sin \omega \Delta t (s-b) \right] \right\}^2 +$$

$$+\left\{\sum_{s=b}^{b+N} K(s-b)\left[U_{s_0}^{s}+U_{s_\omega}^{s}\cos\omega\,\Delta\,t\,(s-b)-U_{s_\omega}^{c}\sin\omega\,\Delta\,t\,(s-b)\right]\right\}^2, \tag{13}$$

where $U_{s_0}^{s}$ and $U_{s_0}^{c}$ are voltages of quadrature components of the video signal at the sth time instant; $U_{s_\omega}^{s}$ and $U_{s_\omega}^{c}$ are the same for the radio pulse.

In the given case, as before, it is assumed that the envelopes of the video and radio signals are identical and their complex amplitudes coincide.

In conclusion it is necessary to note that the class of measuring procedures considered here retains its validity with deviations within reasonable limits of amplitudes, the laws governing the change of envelopes and the relationships of the phases of signals from designed ones. For example, Figs. 2 and 3 show responses of sliding window (8) for initial data (12) with unaccounted for disbalances of amplitudes and the initial phases of radio pulses, respectively. Instead of presumed equality of the indicated parameters, one of the signals had a double amplitude drop (Fig. 2) and the initial phase departure through 30° from nominal ones (Fig. 3).

Together with the increase of the measuring accuracy the approach set out above also improves the ranging resolution in the Rayleigh terms. However, due to the presence of side lobes capable of masking weak signals, narrowing the main lobe of function (8) and (13) more than 4 times with the Gaussian envelope is not desirable. The given measure also makes it possible to restrict the influence on the measuring accuracy of anomalous spikes of ranging estimates beyond response main lobe (8) and (13). If required, a radical increase of resolution can be achieved under conditions of high side lobes F_M based on special procedures of superresolution.

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29 November 1996