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RADIOELECTRONICS AND COMMUNICATIONS SYSTEMS

THE METHOD OF CALCULATING THE RESULTS
OF SUPERESOLUTION IN THE ESTIMATE
OF OTHER PARAMETERS OF SIGNALS

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Pre-discrimination of signals using various parameters is very widely practiced in radar location to increase resolution in any of the coordinates. The effect attainable can be by far greater if we use the procedures of super-resolution. The objective of the article is consideration of the methods of translating the results of super-resolution in the estimate of other signal parameters based on using single-signal measuring algorithms.

Let us consider a problem of determining angular coordinates M of sources of a linear antenna array (LAA) under condition of pre-discrimination of range super-resolution of targets. Let us assume that after analog-to-digital conversion of voltage is effected by accumulating ADC readings within rigidly allocated time intervals – gates. Having the general case as a target let us agree that probing pulses exist during T of such time intervals. For convenience sake let us assume that beginnings of reflected signals are concentrated in one gate. Then, the matrix recording of responses R of LAA spatial channels in T gates of the range can be expressed in the form

$$V = S A F \qquad (1)$$

where V is the matrix of dimension $T \times R$ of complex voltages V_{im} (responses R of LAA of spatial channels in the range T gates); S is the matrix of dimension $T \times M$ of analytical description $S_i (z_m)$ of the responses T of the range gates for M signals and the z_m-shift of the m-pulse in LAA readings with respect to the first of the signal gates; A is the diagonal matrix of the order M, whose main diagonal is formed by the vector of complex amplitudes a_m for M radio pulses; F is the matrix of dimensions $R \times M$ of values $F_i(x_m)$ of complex characteristics of directivity R reception channels in the directions M of the sources (x_m).

Assuming that, as a result of signal resolution in delay time, the components of the matrix S are known, we will determine the matrix of the complex amplitudes of the signals $P = AF$ generalized to the angular coordinate x. Having used to this end the method of the least squares we will express the functional being minimized in the form

$$L = \mathrm{t\,r}\left[\{V - S P\}^{*} \{V - S P\}\right] \qquad (2)$$

where the sign "*" means the complex-conjugate transposition. Hence, as a result of the differentiating of (2) with respect to the matrix P we obtain

$$\frac{\partial L}{\partial P} = -2\{V - S P\}^{*} P \quad \text{and} \quad \hat{P} = \left(S^{*} S\right)^{-1} S^{*} V. \qquad (3)$$

For the material matrix S the sign "*" corresponds to the operation of usual transposition. Its elements also can have a complex nature if the results of the summation of LAA readings in the gates do not coincide in square components.

Having determined according to (3) the generalized signal amplitudes we can solve the range-finding problems applying the known [1, 2] single-signal measuring algorithms. It is very important that in this case the elements of the matrix P corresponding to the unknown source should play the role of voltages. With other locations of the signals in respect to the gate network, it is necessary to introduce changes into the matrix structure S zeroing its respective elements in order

to determine matrix (3). For instance, in situations when the end of one signal falls on the gate of the beginning of the other and the radio pulse length coincides with the gate time length, the matrix S will be linear

$$S = \begin{bmatrix} S_1(z_1) & 0 & 0 & \cdots & 0 \\ S_2(z_1) & S_2(z_2) & 0 & \cdots & 0 \\ 0 & S_3(z_2) & S_3(z_3) & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & S_T(z_m) \end{bmatrix}.$$

The example of the calculating procedure is easily generalized to the problem of joint measuring the time delay and the Doppler frequency; it suffices only to interpret the elements of the matrix F as the values of frequency characteristics R of filters at frequencies M of the sources. Similarly, using the concept of the amplitudes $\dot{Q}_{tm} = \dot{a}_m S_t(z_m)$ generalized over time, an inverse process is implemented, i.e., transition from super-resolution by angular coordinates or frequency to single-signal range finding.

To derive the corresponding calculation procedure let us re-write system of equations (1) in the form

$$V = QF \tag{4}$$

where $Q = SA$ is the matrix of signal amplitude generalized over time. Applicably to the case of the arrival M of radio pulses in one range gate and their existence M during T of the gates, the matrix Q of dimension $T \times M$ will contain elements $a_m S_i(z_m)$.

Using the method of the least squares we obtain for (4) the estimate of the signals generalized over time

$$\hat{Q} = (FP^*)^{-1} F V^*. \tag{5}$$

With the account of the elements of the matrix $\hat{Q}$ necessary for the functioning of the single-signal algorithms it is not difficult to determine the delay time estimates z_m of each of the signals of the package. With the rectangular envelope of the radio pulse whose duration coincides with the gate time length, the respective procedure has the form

$$\hat{z}_m = \frac{|Q_{1m}| N}{|Q_{1m}| + |Q_{2m}|}$$

where z_m is the shift of the m-th pulse in the LAA reading with respect to the end of the first of two gates where there is a signal, $|Q_{1m}|$, $|Q_{2m}|$ are the modules of the amplitudes of the m-th signal in the 1st and 2nd signal gates, N is the gate length in the LAA readings.

It is possible to formulate the gist of the method proposed: the results of super-resolution according to expressions (3) or (5) help us to determine the complex signal amplitudes using the respective parameter and then we use them as voltages within the framework of single-signal measurements. The approach considered makes it possible to lift restrictions which follow from the concept of the indefiniteness function for joint measurement of several parameters. In the given case resolution by one of them simultaneously means resolution by others remaining. It is important to underline that in the algorithms proposed the number of sources determine only the procedure of calculating generalized amplitudes while their calculation in terms of estimates of parameters of concern is done using the same single-signal methods.

REFERENCES

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