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INTERPRETATION OF THE PRONI METHOD FOR SOLVING LONG-RANGE PROBLEMS

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The paper considers problems of the signal super-resolution in terms of delay times based on the Proni method.

Mathematical uniformity of problems dealing with spectral evaluation and determination of time delays of signals creates preconditions for using uniform methods of measuring signal parameters within the framework of procedures of angular direction finding, frequency selection and ranging. The objective of the present article is generalization of the Proni method known in spectral evaluation [1] to the case of super-resolution of narrow-band video- and radio pulses in terms of delay time.

Among all possible models of the envelope of the pulses in this case, only analytical approximations admitting, in the absence of noises, the identical replacement of the sum of voltages M of the signals by the polynomial of power M, will be interesting.

Among such, first of all, it is necessary to point at the bell-like or Gaussian envelope being determined by the function of the form $\exp(-\beta^2 t^2)$. Having in mind the change of ranging directly using the samples of the analog-to-digital converter (ADC) we can neglect insufficiency of physical nature of such a model, which manifests itself in the time function infinity since the evaluation of the delay time can be implemented using the position of the signal maximum. In the case of video pulse the model specified makes it possible to use for their super-resolution the so-called Proni "initial method" (PIM) [1] bringing about the adjustment of M exponents to $2M$ samples of the signal mixture voltages. Let us consider specificity of using it in greater detail.

We will consider shift d_m of the first of $2M$ samples of the digital sampling of voltages with respect to the envelope maximum of the mth pulse as unknown delay time. Let us agree that within the interval ($2M$ samples) all M signals are present and their energy is substantially in excess of variance of noises.

The assumptions made make it possible with the complex nature of the voltages of the signal mixture to express a $2M$-component vector of samples U in the form, free of noises:

$$U = S \cdot A \tag{1}$$

where

$$S = \begin{bmatrix} g^{d_1^2} & g^{d_2^2} & \cdots & g^{d_M^2} \\ g^{(d_1 - z_1)^2} & g^{(d_2 - z_1)^2} & \cdots & g^{(d_M - z_1)^2} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & & \cdot \\ g^{(d_1 - z_{2M-1})^2} & g^{(d_2 - z_{2M-1})^2} & \cdots & g^{(d_M - z_{2M-1})^2} \end{bmatrix},$$

$$g = \exp(-\beta^2 \cdot \Delta t^2)$$

A is the vector of complex amplitudes $\dot{a}_i$ of the signals, Δt is the digitization period, z_{m-1} is the interval between the mth and 1st samples in digitization periods.

In contrast to the exponents in the Proni method [1], the components of the matrix S contain squares of unknown attenuation parameters, where the signal delay times d_m can act on these components. Therefore, the matrix S should be reduced to the form canonical for the IPM form. To this end let us establish an interval between the samples of the measuring sampling as regular, i.e., $z_m = (m-1)\, z_1$, where z_1 is the time interval in discretization periods between the first and second voltage codes used for estimating the signal time delay, m is the ordinal number of the ADC sampling in the measuring sampling, z_m is the distance of the mth sample from the first in the sampling.

Let us introduce a concept of the complex amplitude, generalized to time, of the mth signal $\tilde{a}_m = \dot{a}_m \exp(-\beta^2 \Delta t^2 d_m^2)$ and the vector $\tilde{A} = [\tilde{a}_1\ \tilde{a}_2 \ldots \tilde{a}_M]^T$ of generalized amplitudes corresponding to this signal. In addition, let us open squares in the power signs of the elements g of the matrix S. As a result we will obtain the following equivalent of expression (1):

$$U = \tilde{S} \cdot \tilde{A} \tag{2}$$

where

$$\tilde{S} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \dfrac{y^{2 d_2 z_1}}{y^{z_1^2}} & \dfrac{y^{2 d_1 z_1}}{y^{z_1^2}} & \cdots & \dfrac{y^{2 d_M z_1}}{y^{z_1^2}} \\ \vdots & \vdots & & \vdots \\ \dfrac{y^{2 d_1 (2M-1) z_1}}{y^{(M-1) z_1^2}} & \dfrac{y^{2 d_1 (2M-1) z_1}}{y^{(M-1) z_1^2}} & \cdots & \dfrac{y^{2 d_M (2M-1) z_1}}{y^{(M-1) z_1^2}} \end{bmatrix},$$

$$y = \exp(\beta^2 \Delta t^2).$$

For ruling out the normalizing of the elements of the matrix $\tilde{S}$ it is advisable to go to the weighted vector of the voltage samples

$$\tilde{U} = \left[\dot{U}_1\ \dot{U}_2 y^{z_1^2} \ldots \dot{U}_{2M} y^{(2M-1)^2 z_1^2}\right]^T.$$

In doing so we finally will have

$$\tilde{U} = S_c \cdot \tilde{A} \tag{3}$$

where S_c is the canonical matrix of the real exponents of the IPM form whose elements differ from the elements S by single values of the denominators.

Using this model of measuring sampling it remains to use the Proni procedure for adjusting M real exponents to $2M$ complex samplings of the data. At the first of its stages it is necessary [1] to determine the M-component vector of the coefficients B of the polynomial of power M having solved the matrix equation:

$$C \cdot B = X \tag{4}$$

where in the given case

$$C = \begin{bmatrix} \dot{U}_M y^{(M-1)^2 z_1^2} & \dot{U}_{M-1} y^{(M-2)^2 z_1^2} & \cdots & \dot{U}_1 \\ \dot{U}_{M+1} y^{M^2 z_1^2} & \dot{U}_M y^{(M-1)^2 z_1^2} & \cdots & \dot{U}_2 y^{z_1^2} \\ \vdots & \vdots & \cdots & \vdots \\ \dot{U}_{2M-1} y^{(2M-2)^2 z_1^2} & \dot{U}_{2M-2} y^{(2M-3)^2 z_1^2} & \cdots & \dot{U}_M y^{(M-1)^2 z_1^2} \end{bmatrix},$$

M

$$X = -\left[\dot{U}_{M+1}\, y^{M^2 z_1^2}\; \dot{U}_{M+2}\, y^{(M+1)^2 z_1^2} \ldots \dot{U}_{2M}\, y^{(2M-1)^2 z_1^2}\right]^T.$$

$$B = [b_1\, b_2 \ldots b_M]^T.$$

The matrix of the samples in the left-hand part of Eq. (4) is the Toeplitz matrix, therefore for its solution it is convenient to use fast computational algorithms [2].

The evaluation of the vector B can be found also using the method of the least squares:

$$B = (C^* C)^{-1} \cdot C^* \cdot X \tag{5}$$

where * is the sign of complex conjugate.

Having determined in this way the components of the vector B, at the other stage it is necessary to calculate the roots of the polynomial of power M:

$$\sum_{m=0}^{M} b_m \cdot y^{M-m} = 0 \tag{6}$$

under condition that $b_0 = 1$ (here $\tilde{y} = y^{2dz_1}$). Estimates M of unknown $\tilde{y}_m$ obtained as a result of factorization (6) make it possible to get sought-for values of signal delay times:

$$d_m = \frac{1}{2 z_1 \beta^2 \Delta t^2} \ln |\tilde{y}_m|. \tag{7}$$

Let us synthesize a one-signal measuring procedure as an option of practical implementation of the given approach. From (4) for the case of the single source we obtain: $\dot{U}_1 \cdot b_1 = -\dot{U}_2 \cdot y^{z_1^2}$. A respective version of polynomial (6) will take on the form: $b_0 \tilde{y} + b_1 = \tilde{y} - \dfrac{\dot{U}_2 y^{z_1^2}}{\dot{U}_1} = 0$, hence $d_1 = \dfrac{1}{2\beta^2 \Delta t^2 z_1} \ln \dfrac{|\dot{U}_2|}{|\dot{U}_1|} + \dfrac{z_1}{2}$.

When using the method of the least squares the evaluation of the coefficient b_1 will become more complicated, but taking into account that

$$U_1^s U_2^c - U_1^c U_2^s = |\dot{U}_1| \cdot |\dot{U}_2| (\sin\varphi \cos\varphi - \cos\varphi \sin\varphi) = 0$$

where φ is the signal initial phase, the sought-for estimate of the ranging counter to anticipation will be cumbersome as well:

$$d_1 = \frac{1}{2\beta^2 \Delta t^2 z_1} \ln \frac{U_1^c U_2^c + U_1^s U_2^s}{U_1^{c\,2} + U_1^{s\,2}} + \frac{z_1}{2}.$$

Let us note that it is not difficult to arrive to the same formulas for d_1 also based on the particular voltages of the pair of ADC samples. In the two-signal case similar calculations for (4) and (6) are reduced to the following:

$$\begin{bmatrix} \dot{U}_2 y^{z_1^2} & \dot{U}_1 \\ \dot{U}_3 y^{4 z_1^2} & \dot{U}_2 y^{z_1^2} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = -\begin{bmatrix} \dot{U}_3 y^{4 z_1^2} \\ \dot{U}_4 y^{9 z_1^2} \end{bmatrix};$$

$$\tilde{y}^2 + b_1 \tilde{y} + b_2 = 0,$$

and in this case estimates b_1, b_2 by virtue of the least squares are as follows:

$$b_1 = \frac{(\tilde{U}_{12} + \tilde{U}_{23})(\tilde{U}_{13} + \tilde{U}_{42}) - (\tilde{U}_1^2 + \tilde{U}_2^2)(\tilde{U}_{23} + \tilde{U}_{43})}{\text{Det}};$$

$$b_2 = \frac{(\tilde{U}_{23} + \tilde{U}_{43})(\tilde{U}_{12} + \tilde{U}_{23}) - (\tilde{U}_1^2 + \tilde{U}_3^2)(\tilde{U}_{13} + \tilde{U}_{42})}{\text{Det}};$$

$$\text{Det} = (\tilde{U}_1^2 + \tilde{U}_3^2)(\tilde{U}_1^2 + \tilde{U}_2^2) - (\tilde{U}_{12} + \tilde{U}_{23})^2,$$

$$\tilde{U}_{nm} = \tilde{U}_n^c \tilde{U}_m^c + \tilde{U}_n^s \tilde{U}_m^s,\ \tilde{U}_n^2 = \tilde{U}_n^{c\,2} + \tilde{U}_n^{s\,2},$$

$$\tilde{U}_2 = \dot{U}_2 y^{z_1^2},\ \tilde{U}_3 = \dot{U}_3 y^{4 z_1^2},\ \tilde{U}_4 = \dot{U}_4 y^{9 z_1^2}.$$

The version of IMP based on the real components can be used also for super-resolution of complex Gaussian radio pulses. To this end it is necessary, so to say, preliminary finalize all phase ADC samples by a value compensating its misadjustment over a time interval between them. The respective procedure of phasing the elements of quadratures of the weighted vector of the samples $\tilde{U}$ will be written in the form:

$$\tilde{U}_{s\,\mathrm{ph}}^c = \tilde{U}_s^c \cos p_s + \tilde{U}_s^s \sin p_s;\quad \tilde{U}_{s\,\mathrm{ph}}^s = \tilde{U}_s^s \cos p_s - \tilde{U}_s^c \sin p_s$$

where $p_s = \omega\, \Delta t\, z_s$, z_s is the time interval between the first and the sth complex voltages of the signals.

When using the adjustment of the data for the complex exponents the necessity in the preliminary increase of the phase of the samples of the radio pulses is ruled out. In this case the model itself of using the sampling in (3), where each mth element of the nth row of the matrix S_c is additionally multiplied by the value $\exp\,[j\, \omega_m\, \Delta\, t\, z_1\, (n-1)]$, where ω_m is the known frequency of filling the mth radio pulse, is subjected to the change.

As a conclusion, factorization of the polynomial corresponding to expression (3) will be conducted with respect to the complex roots

$$\tilde{y}_m = \exp\,[\beta^2\, \Delta\, t^2\, 2\, d_m\, z_1 + j\, \omega_m\, \Delta\, t\, z_1\,]$$

while the unknown delay times are determined through their moduluses $|\,\tilde{y}_m\,|$ by virtue of formula (7).

With the approach considered it is possible to estimate instances of signal arrivals and their filling frequencies jointly if the latter are known. Bearings of the sources can also play the role of the frequencies. In this case the sought-for estimates of the accompanying delay time of the parameter are determined according to [1] with respect to the quadratures of the complex roots of the polynomial $\tilde{y}_m$. In this case

$$\omega_m = \mathrm{arctg}\,[\mathrm{Im}\,\{\tilde{y}_m\} \,/\, \mathrm{Re}\,\{\tilde{y}_m\,\}\,] \,/\, (\Delta\, t\, z_1).$$

It is characteristic that resolution in the delay time and accuracy of its measurement do not depend on the presence and quality of information on the frequency or the bearing of the signal.

Since a priori information on the number of sources of the signal normally are absent, the use of the Proni method is closely linked with the determination of the order of the model M. The given task can be solved using the approaches traditional for it and associated with the analysis of the singular numbers of the matrix of samples [1] or with the artificial increase in the number of signals and screening of noise solutions based on linear prediction: forward – backward [1]. A more compact alternative in terms of computational volumes is in using a modified probability function from which unknown amplitudes of the signals are excluded [3, 4]. In this case, within the framework of the Proni method all admissible models of the signal mixture with the ascending order from 1 to M are adjusted, while the estimates obtained of the delay times of the signals are then substituted into a similar [4] modification of the probability function:

$$F_{\text{M}} = -\frac{D}{D_{\text{M}}} = \max \tag{8}$$

where D_{M} is the matrix determinant $[\dot{Q}_{ij}\,]$, $i, j = 1, \ldots, M$; D is the matrix determinant, which is different from $[\dot{Q}_{ij}\,]$ by the first row and column with the elements $0, \dot{W}_1^*, \ldots, \dot{W}_{\text{M}}^*$;

$$\dot{W}_n = \sum_{s=r_n}^{r_n+N-1} (U_s^c + j\, U_s^s)\, \dot{K}_{sn}^*;\ \dot{Q}_{mn} = \dot{Q}_{nm}^* = \sum_{s=r_m}^{r_n+N-1} \dot{K}_{sm}\, \dot{K}_{sn}^*,$$

$$\dot{K}_{sm} = K_{sm}\, \{\cos\,(\omega_m\, \Delta\, t\, (s - r_m) + j \sin\,(\omega_m\, \Delta\, t\, (s - r_m))\,\},$$

$$\dot{K}_{sm}^* = K_{sm}\, \{\cos\,(\omega_m\, \Delta\, t\, (s - r_m) - j \sin\,(\omega_m\, \Delta\, t\, (s - r_m))\,\},$$

s is the current number of a time sample, r_n is the number of the first among time samples within the limits of the existence of the nth signal, $r_n = s_{n_{\max}} - d_n$, $s_{n_{\max}}$ is the position of the nth signal maximum, d_n is the estimate of time delay (7) of the nth pulse,

$$K_{sm} = \begin{cases} \exp\left(-\beta^2 \Delta t^2 \left(s - r_m - \frac{N}{2}\right)^2\right) \text{with } r_m \leq s \leq r_m + N \\ 0 \text{ at } s < r_m \text{ and } s > r_m + N; \end{cases}$$

N is the signal length in the ADC samples (it is assumed that all signals have the same time length).

In the case of video pulses instead of $\dot{K}_{sm}$ and $\dot{K}^*_{sm}$ the real function K_{sm} should be used directly instead of K_{sm} and K^*_{sm} .

It is essential that the dimension of the vector of the samples of voltages U can be limited by the order M. In this case the set of estimates r_m maximizing criterion (8) is most likely.

In conclusion it should be noted that different modifications of the Proni method does not exhaust all possible approaches to the solution of the ranging super-resolution problems based on deterministic models of signals. In addition to such advantages as high accuracy in estimating delay times, its independence of the accuracy of determining frequency or bearing of the signals, the Proni method is characterized by a setback linked with the necessity of processing $2M$ complex samples of the data. Such dimension of measuring sampling is redundant at least by the fact that for the resolution of M pulses with the unknown arrival times and complex amplitudes it is sufficient to have 1.5 M complex samples (with even M) or $1.5\,M + 0.5$ (with M odd). However, if we conduct separation of unknown amplitudes of the signals, then for reducing the range-finding problem to finding the roots of the polynomial of the power M with the Gaussian envelope we will need only M+1 complex voltage.

In the case of the digitization limited frequency and small lengths of the signals the above factor can become instrumental when choosing other approaches not so critical to the length of the voltage sampling being processed.

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