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MEASURING THE ADC DIGITIZATION BY THE SUM OF HARMONIC EFFECTS

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The paper considers the procedures for measuring the ADC digitization period based on the use of the sum of harmonic oscillations of different frequencies as a test signal; the paper also provides respective analytical estimates of accuracy.

When using methods of measuring the digitization period Δt [1], as one method of increasing the validity of the estimates obtained we recommend the consecutive measurement Δt in terms of harmonic signals of different frequencies. Such an approach makes it possible to average the estimation errors determined by nonlinear distortions of harmonic signals in the oscillatory device, especially an analog one, under the condition that said nonlinear effects depend on the frequency of oscillations generated. After a number of experiments conducted in the way indicated above, we had the idea of reducing the estimation time Δt by feeding to the ADC input simultaneously a number of harmonic signals. The objective of this article is consideration of corresponding algorithms of measuring the ADC digitization period and the analysis of their potential accuracy.

When using standard analog oscillators as sources of test signals it does not seem possible to attain an identical initial phase of harmonic effects of different frequencies in the summary signal. Therefore, it is preferable to begin the synthesis of the measuring procedure based on the condition of accounting for the difference in the phases of harmonic oscillations.

Having used the method of maximum likelihood in the assumption of non-correlation and the Gaussian nature of noises we will write down the information equivalent of the likelihood function for the real form of representing the signals:

$$F = \sum_{s=1}^{S} \left\{ U_s - \sum_{m=1}^{M} a_m \cos(\omega_m \Delta t (s-1) + \varphi_m) \right\}^2$$

or having denoted

$$a_m^c = a_m \cos \varphi_m, \quad a_m^s = a_m \sin \varphi_m, \quad p_{ms} = \omega_m \Delta t (s-1),$$

$$F = \sum_{s=1}^{S} \left\{ U_s - \sum_{m=1}^{S} (a_m^c \cos p_{ms} - a_m^s \sin p_{ms}) \right\}^2 = \min. \tag{1}$$

Using the approach considered in [1] the sought-for measuring procedure is not difficult to represent as

$$F_M = \sum_{m=1}^{M} \left[a_m^c \sum_{s=1}^{S} U_s \cos p_{ms} - a_m^s \sum_{s=1}^{S} U_s \sin p_{ms} \right] = \max, \tag{2}$$

where a_m^c, a_m^s are the estimates of quadrature components of the amplitude of the mth signal.

Here it is taken into account that the minimum of expression (1) is achieved with the maximum of the squared double products. In addition, an assumption was made that the amplitude of the signals over the number S of the samples remained invariant.

As for the estimates of the amplitudes they, first of all, can be obtained solving the system of likelihood equations:

$$\partial F / \partial a_m^c = 0; \;\; \partial F / \partial a_m^s = 0.$$

Based on Cramer's rule, as a result, one may write down:

$$a_m^c = D_m^c / D \, ; \;\; a_m^s = D_m^s / D, \tag{3}$$

where

$$D = \begin{vmatrix} B_{11} & C_{12} & \cdots & C_{1M} \\ C_{21} & B_{22} & \cdots & C_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ C_{M1} & C_{M2} & \cdots & B_{MM} \end{vmatrix}, \tag{4}$$

$$B_{nn} = \begin{bmatrix} \sum\limits_{s=1}^{S} \cos^2 p_{ns} & -0{,}5 \sum\limits_{s=1}^{S} \sin 2p_{ns} \\ -0{,}5 \sum\limits_{s=1}^{S} \sin 2p_{ns} & \sum\limits_{s=1}^{S} \sin^2 p_{ns} \end{bmatrix},$$

$$C_{nm} = \begin{bmatrix} \sum\limits_{s=1}^{S} \cos p_{ns} \cos p_{ms} & -\sum\limits_{s=1}^{S} \cos p_{ns} \sin p_{ms} \\ -\sum\limits_{s=1}^{S} \cos p_{ms} \sin p_{ns} & \sum\limits_{s=1}^{S} \sin p_{ns} \sin p_{ms} \end{bmatrix}, \; C_{nm} = C_{mn}^{\mathrm{T}},$$

determinants D_m^c, D_m^s are obtained from the determinant D by the replacement of the respective mth even (for D_m^c) or the odd (for D_m^s) column with the vector of free terms $\{W_1, W_2, W_3 \dots W_M\}^{\mathrm{T}}$, where

$$W_m = \begin{bmatrix} -\sum\limits_{m=1}^{S} U_s \cos p_{ms} & \sum\limits_{s=1}^{S} U_s \sin p_{ms} \end{bmatrix}^{\mathrm{T}}.$$

Thus, the problem of determining Δt by the sum M of harmonic signals represented in real form was reduced to maximization (2) taking into account (3) by the exhaustive search for possible values Δt until the moment function (2) attains the global maximum.

Let us estimate the potential accuracy of the given method using the Cramer–Rao lower boundary. To reduce calculations let us resort to the matrix notation, having represented the vector of samples as:

$$U = FA, \tag{5}$$

where $U = [\, U_1 \;\; U_2 \;\cdots\; U_S \,]^{\mathrm{T}}$ is the vector of voltages of the signal mixture; $A = [\, a_1^c \;\; a_1^s \;\; a_2^c \;\; a_2^s \;\cdots\; a_M^c \;\; a_M^s \,]^{\mathrm{T}}$ is the vector of quadrature components of the amplitudes of the signals,

$$F = \begin{bmatrix} \cos p_{11} & -\sin p_{11} & \cos p_{21} & -\sin p_{21} & \cdots & \cos p_{M1} & -\sin p_{M1} \\ \cos p_{12} & -\sin p_{12} & \cos p_{22} & -\sin p_{22} & \cdots & \cos p_{M2} & -\sin p_{M2} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \cos p_{1S} & -\sin p_{1S} & \cos p_{2S} & -\sin p_{2S} & \cdots & \cos p_{MS} & -\sin p_{MS} \end{bmatrix}.$$

Using the expression of the Fischer matrix obtained in [2] based on Neudekker applicable to the case under consideration we will write down:

$$I=\frac{1}{\sigma_n^2}\begin{bmatrix} F^T F & (A^T\otimes F^T)\dfrac{\partial F}{\partial \Delta t} \\ \left(\dfrac{\partial F}{\partial \Delta t}\right)^T (A\otimes F) & \left(\dfrac{\partial F}{\partial \Delta t}\right)^T (A A^T\otimes 1_S)\dfrac{\partial F}{\partial \Delta t}\end{bmatrix}, \tag{6}$$

where the elements of the block $F^T F$ are identical to the elements of determinant (4),

$$\left\{(A^T\otimes F^T)\frac{\partial F}{\partial \Delta t}\right\}^T=\left(\frac{\partial F}{\partial \Delta t}\right)^T (A\otimes F),$$

$$(A^T\otimes F^T)\frac{\partial F}{\partial \Delta t}=\begin{bmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ T_M\end{bmatrix},\quad T_n=\begin{bmatrix} -\sum\limits_{s=1}^{S}\sum\limits_{m=1}^{M} a_m\, p'_{m\,s}\cos p_{n\,s}\sin(p_{m\,s}+\varphi_m) \\ \sum\limits_{s=1}^{S}\sum\limits_{m=1}^{M} a_m\, p'_{m\,s}\sin p_{n\,s}\sin(p_{m\,s}+\varphi_m)\end{bmatrix},$$

$$\left(\frac{\partial F}{\partial \Delta t}\right)^T (A A^T\otimes 1_S)\frac{\partial F}{\partial \Delta t}=\sum_{s=1}^{S}\left\{\sum_{m=1}^{M} a_m\, p'_{m\,s}\sin(p_{m\,s}+\varphi_m)\right\}^2 .$$

The estimate of variance $\sigma_{\Delta t}^2$ we are interested in may be found as a result of inversion of matrix (6). Taking into account the known mechanism of matrices one may write down:

$$\sigma_{\Delta t}^2\geq\frac{\sigma_n^2\,|F^T F|}{\begin{vmatrix} F^T F & \cdot & (A^T\otimes F^T)\dfrac{\partial F}{\partial \Delta t} \\ \cdots & & \cdots \\ \left(\dfrac{\partial F}{\partial \Delta t}\right)^T (A\otimes F) & \cdot & \left(\dfrac{\partial F}{\partial \Delta t}\right)^T (A A^T\otimes 1_S)\dfrac{\partial F}{\partial \Delta t}\end{vmatrix}} . \tag{7}$$

In the case of one source

$$F^T F=\begin{bmatrix} \sum\limits_{s=1}^{S}\cos^2 p_{n\,s} & -0{,}5\sum\limits_{s=1}^{S}\sin 2p_{n\,s} \\ -0{,}5\sum\limits_{s=1}^{S}\sin 2p_{n\,s} & \sum\limits_{s=1}^{S}\sin^2 p_{n\,s}\end{bmatrix},$$

$$\left(\frac{\partial F}{\partial \Delta t}\right)^T (A A^T\otimes 1_S)\frac{\partial F}{\partial \Delta t}=\sum_{s=1}^{S}\{a_1\, p'_{1s}\sin(p_{1s}+\varphi_1)\}^2,$$

$$(A^T\otimes F^T)\frac{\partial F}{\partial \Delta t}=\begin{bmatrix} -\sum\limits_{s=1}^{S} a_1\, p'_{1\,s}\cos p_{1\,s}\sin(p_{1\,s}+\varphi_1) \\ \sum\limits_{s=1}^{S} a_1\, p'_{1\,s}\sin p_{1\,s}\sin(p_{1\,s}+\varphi_1)\end{bmatrix}.$$

Hence, as a result of simple transformations one may obtain the known [1] estimate of the variance of the measuring error Δt in terms of the harmonic signal.

Note that for the formation of the multi-frequency package it is not so necessary to have several oscillators, it is enough to successively store in the files the arrays of the samples M of harmonic oscillations obtained as a result of digitization and then add them according to the ordinal number of the samples. The resultant file will be the sum of M test signals. The only drawback of such an approach is the growth of the noise variance resulting from summation. However, it is easily compensated by the increase of the sampling dimension. In addition, given negative cross correlation of oscillator noises at different frequencies it may happen that the resultant noise power will drop.

Another approach, free of the problem of the growth of the noise total variance, is reduced to the use of the program simulation of the sum of the signals of M frequencies with its subsequent reproduction in an analog form by means of DAC. In this case the test oscillator or their set are replaced by one more computer with the board of the digital-analog converter. Given DAC high capacity (14 or more bits) and sufficient high speed one may attain the comparatively low value of digital noise variance. Therefore such an approach for low-frequency ADC may be preferable.

As was noted in [1] given the real form of representing the signal the variance of the estimate Δt depends on the initial phase of the test effect, therefore the transition to the complex representation of voltages, for instance, by discrete Gilbert filtration is advisable. In the case of M signals the given recommendation still remains valid, corroboration of which may be complex alternative (2) and the accuracy estimate corresponding to it.

Let us write down the functional minimized in the form:

$$F=\sum_{s=1}^{S}\left\{U_s^c-\sum_{m=1}^{M}a_m\cos\left(\omega_m\Delta t\,(s-1)+\varphi_m\right)\right\}^2+$$
$$+\sum_{s=1}^{S}\left\{U_s^s-\sum_{m=1}^{M}a_m\sin\left(\omega_m\Delta t\,(s-1)+\varphi_m\right)\right\}^2.$$

Using designations of (1) it will be not difficult to obtain complex analog (2):

$$F_M=\sum_{m=1}^{M}a_m^c\sum_{s=s_1}^{S}\left\{U_s^c\cos p_{sm}+U_s^s\sin p_{sm}\right\}+$$
$$+\sum_{m=1}^{M}a_m^s\sum_{s=s_1}^{S}\left\{U_s^s\cos p_{sm}-U_s^c\sin p_{sm}\right\}=\max. \qquad (8)$$

The corresponding estimate of the variance may be formed from (7) by the substitution of the complex conjugate symbol for the transposition sign "T". In this case it is necessary to take into account that the components of matrix expression (5) will take the form

$$U=[\dot U_1\ \dot U_2\ \ldots\ \dot U_S]^{\mathrm T},A=[\dot a_1\ \dot a_2\ \ldots\ \dot a_M]^{\mathrm T},$$
$$F=\begin{bmatrix}\exp(j\,p_{11}) & \exp(j\,p_{21}) & \cdots & \exp(j\,p_{M1})\\ \exp(j\,p_{12}) & \exp(j\,p_{22}) & \cdots & \exp(j\,p_{M2})\\ \vdots & \vdots & \vdots & \vdots\\ \exp(j\,p_{1S}) & \exp(j\,p_{2S}) & \cdots & \exp(j\,p_{MS})\end{bmatrix},$$

as a result of which we will obtain:

$$F^* F = \begin{bmatrix} S & \cdots & \sum_{s=1}^{S} \exp[j(p_{Ms} - p_{1s})] \\ \vdots & \cdots & \vdots \\ \sum_{s=1}^{S} \exp[j(p_{1s} - p_{Ms})] & \cdots & S \end{bmatrix}$$

$$\left(\frac{\partial F}{\partial \Delta t}\right)^* (A A^* \otimes 1_S) \frac{\partial F}{\partial \Delta t} = \sum_{s=1}^{S} \left\{ \sum_{m=1}^{M} \dot{a}_m p'_{ms} \exp(-j p_{ms}) \right\} \times$$

$$\times \left\{ \sum_{m=1}^{M} a_m^* p'_{ms} \exp(j p_{ms}) \right\},$$

$$(A^* \otimes F^*) \frac{\partial F}{\partial \Delta t} = \begin{bmatrix} \sum_{s=1}^{S} \sum_{m=1}^{M} j a_m^* p'_{ms} \exp[j(p_{ms} - p_{1s})] \\ \sum_{s=1}^{S} \sum_{m=1}^{M} j a_m^* p'_{ms} \exp[j(p_{ms} - p_{2s})] \\ \vdots \\ \sum_{s=1}^{S} \sum_{m=1}^{M} j a_m^* p'_{ms} \exp[j(p_{ms} - p_{Ms})] \end{bmatrix}.$$

Given a single harmonic signal $M = 1$ and

$$F^* F = S, \ (A^* \otimes F^*) \frac{\partial F}{\partial \Delta t} = j a_1^* \sum_{s=1}^{S} p'_{1s},$$

$$\left(\frac{\partial F}{\partial \Delta t}\right)^* (A^* \otimes F) = -j a_1 \sum_{s=1}^{S} p'_{1s},$$

$$\left(\frac{\partial F}{\partial \Delta t}\right)^* (A A^* \otimes 1_S) \frac{\partial F}{\partial \Delta t} = \sum_{s=1}^{S} a_1^2 p'^2_{1s}.$$

Hence,

$$\sigma^2_{\Delta t} \geq \frac{\sigma_n^2}{a_1^2 \left(\sum_{s=1}^{S} p'^2_{1s} - \frac{1}{S} \left(\sum_{s=1}^{S} p'_{1s} \right)^2 \right)}.$$

Taking into account that $p'_{1s} = \omega_1 (s - 1)$ after a number of transformations we may obtain:

$$\sigma^2_{\Delta t} \geq \frac{12\sigma_n^2}{a_1^2 \omega_1^2 S (S^2 - 1)}, \tag{9}$$

which is in good conformity with known relationships for the real test signal [1].

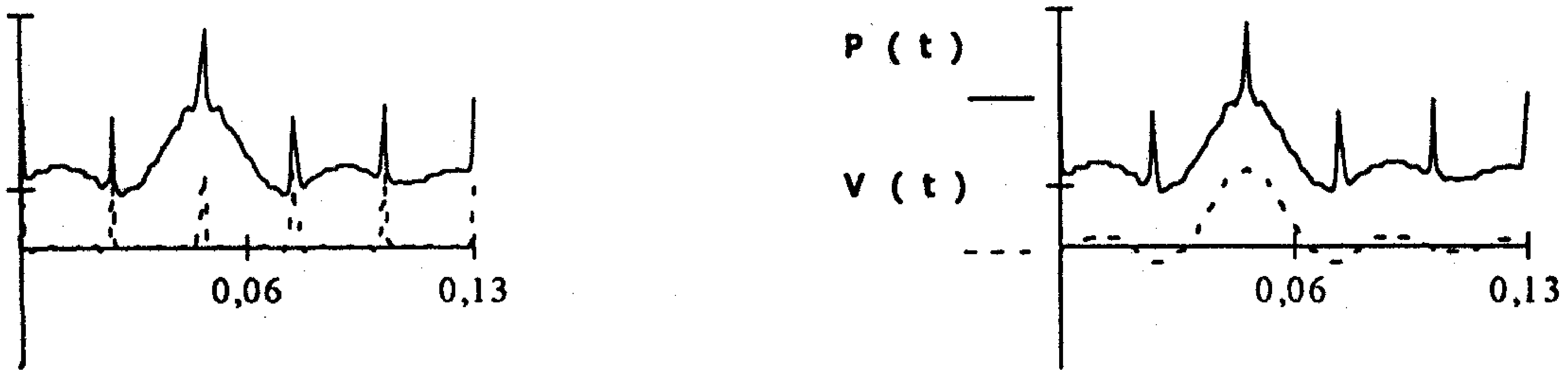

Fig. 1

Thus, the relationship between $\sigma^2_{\Delta t}$ and the initial phase given the single signal disappeared, q.e.d. Given the large number of complex harmonic effects, it is not the initial phases of the signal which are important, but rather their differences. One may become sure of this having analyzed the arguments of the complex exponents of the Fischer information matrix determinant. In particular, given $M = 2$ the lower element of its main diagonal has the form

$$\left(\frac{\partial F}{\partial \Delta t}\right)^* (A A^* \otimes 1_s) \frac{\partial F}{\partial \Delta t} =$$

$$= \sum_{s=1} \left(\sum_{m=1}^{M} a_m^2 \, p'^2_{ms} + 2a_1 a_2 \, p'_{1s} \, p'_{2s} \cos(\Delta \omega_{12} \Delta t (s-1) - \Delta \varphi_{12}) \right),$$

where $\Delta\omega_{12} = \omega_1 - \omega_2$, $\Delta\varphi_{12} = \varphi_1 - \varphi_2$. Other transcendental components obtained as a result of the opening and subsequent transformations of denominator determinant (7) depend on values $\Delta\omega_{12}$ and $\Delta\varphi_{12}$.

To investigate the nature of relationship $\sigma^2_{\Delta t}$ by means of Mathcad package 7.0 a calculation experiment applicable to $M = 2$ was conducted. In this case we considered different relationships of the amplitudes, frequencies of the signals, and their initial phases and the value of product $\Delta\omega_{12}\,\Delta t$ was varied. As a result of calculating variance (9) and value $\sigma^2_{\Delta t}$ corresponding to the two-signal situation it was established that superiority in accuracy over the one-signal measuring procedure within the limits of its zone of single-valued measurement is ensured by the two-frequency approach only for a limited number of conditions. In this case the greatest advantage in the error variance in the case of favorable parameters of the signals does not exceed two. It is essential that the relationships of values $\Delta\omega_{12}$ and Δt under other equal conditions may radically change the situation having completely deprived two-signal estimation of any advantages. Therefore, if the maximum frequency of the signals generated does not go beyond the limits of the band corresponding to single-valued measurement Δt in terms of unit input the application of two-signal estimation may be considered only as an additional procedure with respect to the one-signal method. This means that the estimate Δt obtained at the stage of monosignal measurement may be refined even further having carefully chosen the parameters of the two-frequency mixture.

As for the explicit preferable nature of multi-signal measurements, one may seriously speak about it only when in the test package one uses oscillations exceeding the above-mentioned limit of single-valuedness. In this case a substantial difference in variances $\sigma^2_{\Delta t}$ constituting several orders is achieved and the optimal combination of all parameters of the multi-frequency package is advisable in this case. Figure 1 shows maximization results in the Mathcad 7.0 package of relationship (8) in the one- (curve $V(t)$) and two-signal (curve $P(t)$) cases given noise zero variance. Value 0.05 corresponds to true value Δt on the horizontal axis of the plot. In this case Fig. 1a for one-signal measurements used a harmonic oscillation of high frequency, while Fig. 1b that of lower frequency. It is not difficult to notice that the zone of the one-valued measurement using the two-signal method substantially exceeds a similar band for the one-signal approach. The width of the main peak of the estimates Δt of the functions maximized by exhaustive search provides a judgement about advantage in accuracy.

It should be noted that the estimates of the amplitudes used in (2) and (8) may be obtained also as a result of the fast Fourier transform (FFT) when using the recommendations of [3].

In conclusion we have to apologize to our readers for the misprint, made through our fault, in reference [1] mentioned above: the correct notation of expression (9) on page 46 [1] should be as follows:

$$Q_M = \frac{D_1^2 f_2 + D_2^2 f_1 - D_1 D_2 f_3}{D} = \max.$$

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