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VOLUME 47 **NUMBER 2** **2004**

RADIOELECTRONICS AND COMMUNICATIONS SYSTEMS

Radioelectronics and Communications Systems
Vol. 47, No. 2, pp. 30–35, 2004

Izvestiya VUZ. Radioelektronika
Vol. 47, No. 2, pp. 42–50, 2004
UDC 621.39

A METHOD FOR CORRECTION OF QUADRATURE DISBALANCE OF RECEPTION CHANNELS IN A DIGITAL ANTENNA ARRAY

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The paper describes a new method for correction of quadrature nonidentities of reception channels in a digital antenna array. The method is distinguished by minimal computation.

The realization of the potential of the digital RP-shaping in radiolocation, cellular communication systems of the third and four generation, and in other radio-engineering applications requires, as a necessary condition, minimization of hardware errors which are inevitable at signal processing. One of the ways to resolve this problem is the application of special procedures to correct the receiver channels' characteristics, since their different transfer factors inevitably lead to the distortion of signal arrays and, consequently, to the loss of information. During the last few years this issue has been investigated extensively [1]. However, regardless of the structure of the receivers coupled with the antenna elements (non-quadrature or quadrature-type with orthogonal subchannels), the development of correction procedures for reception channel characteristics in digital antenna arrays (DAA) was mainly related to compensation of their inter-channel discrepancies in the amplitude- and phase-frequency responses. At the same time, in the case of analog orthogonalization of signals (one of its implementations is presented in Fig. 1), the procedures of correction of inter-channel nonidentities cannot eliminate completely the impact of different gain factors of the quadrature subchannels, or neutralize their non-orthogonality. It is possible only if using special procedures for correction of quadrature disbalance [2]. Meanwhile, the known procedures of this type are distinguished with a large amount of computations, which complicates the realization in real time at high-frequency quantization of signals.

With regard for the fact that the potentialities of the quadrature algorithm of processing (Fig. 1) outperform considerably the non-quadratic reception scheme [2] when working with broad-band signals, the purpose of this paper is the development of a method of correction of quadrature subchannels' nonidentities in the case of analog orthogonalization of signals in the reception path of digital antenna arrays. The new method must be distinguished with moderate hardware expenditures at its implementation.

It is known that in receivers with orthogonal subchannels the main contributors of errors are nonidentities of gain factors of quadrature channels, and deviation from $\pi/2$ of the phase shift introduced into one of the subchannels. The phase error may also include a spread in the group delay of signals.

Prior to the derivation of the correction algorithm, assume that the check signal (CS) represents a continuous harmonic oscillation, whose period T_0 is related to the ADC clock period:

$$T_{\text{ADC}} = T_0 (2n+1)/4, \; n = 0, 1, 2, \dots . \tag{1}$$

If one of the subchannels is taken as a reference channel, with its output voltage

$$a_k(i) = a_s \sin(2\pi f_s i T_{\text{ADC}} + \varphi_s), \tag{2}$$

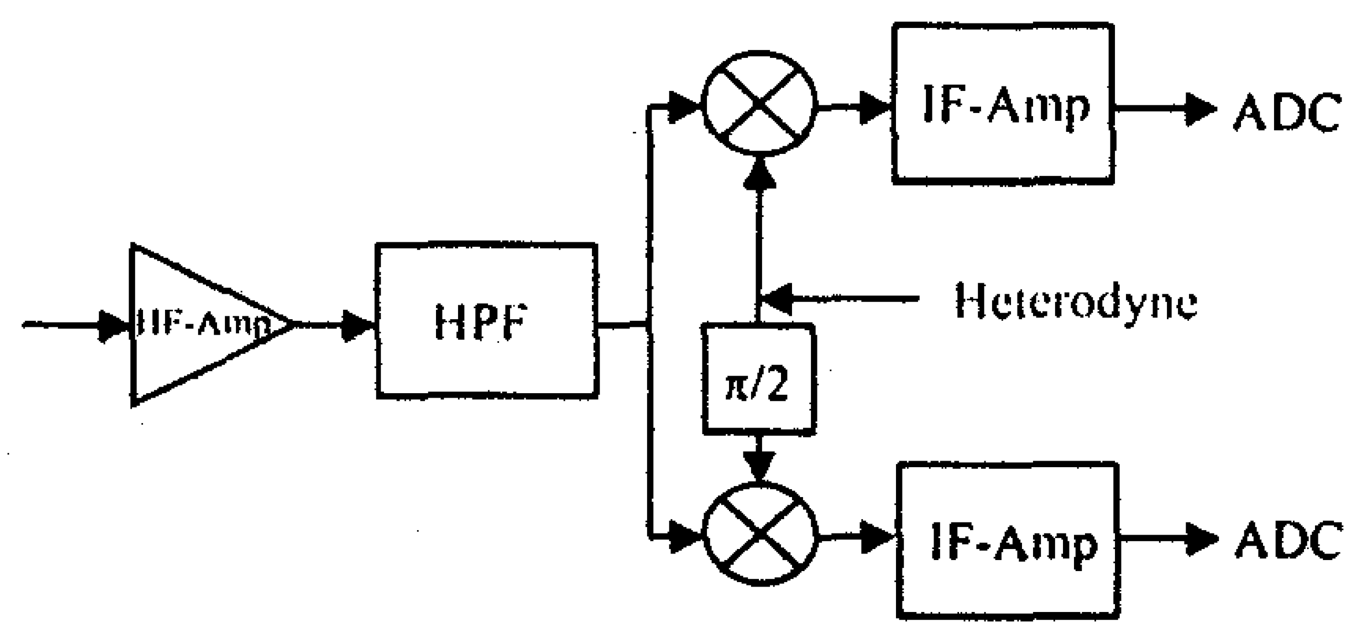

Fig. 1

Table 1

Δ°	−18	−9	−4.5	0	4.5	9	18
U^C	26.949	28.424	29.038	29.564	29.999	30.34	30.733
U^S	−16.514	−14.483	−13.387	−12.246	−11.067	−9.858	−7.378
U	31.606	31.901	31.975	32	31.975	31.901	31.606
ΔU,%	−1.23	−0.31	−0.078	0	−0.078	−0.31	−1.23
Δψ	−9	−4.5	−2.25	0	2.25	4.5	9

Table 2

$(1+\Delta A)$	0.7	0.8	0.9	1	1.1	1.2	1.3
ΔA,%	−30	−20	−10	0	10	20	30
U^C	25.13	26.608	28.086	29.564	31.042	32.521	33.999
U^S	−10.409	−11.021	−11.634	−12.246	−12.858	−13.47	−14.083
U	27.2	28.8	30.4	32	33.6	35.2	36.8
ΔU,%	−15	−10	−5	0	5	10	15
Δψ	~0	~0	~0	~0	~0	~0	~0

where k is the ordinal number of a spatial channel; $i = 0, 1, 2,..., I-1$, I is the number of time samplings of ADC; a_s and φ_s are the signal amplitude and initial phase, respectively; f_s is the pilot signal frequency; and T_{ADC} is the ADC clock period, then the relative nonidentities of transfer factors of the quadrature subchannels can be mapped in samples of the second subchannel in the following manner:

$$b_k(i) = (1+\Delta A)a_s \sin\left(2\pi f_s i T_{ADC} + (\Delta + \pi/2) + \varphi_s\right) \quad (3)$$

where Δ is the phase error of "dequadratization", and ΔA is the absolute value of the amplitude distortion.

The impact of intrachannel errors on orthogonal components of the complex voltage amplitude of the kth spatial channel during the ith time sample has been checked with the aid of a program developed in the MathCad medium and generating signals' voltages by algorithms (2) and (3). The program also performs digital filtering of these signals using the method of additional gating of ADC samples [3]. The investigation results are presented in Tables 1 and 2. The first table

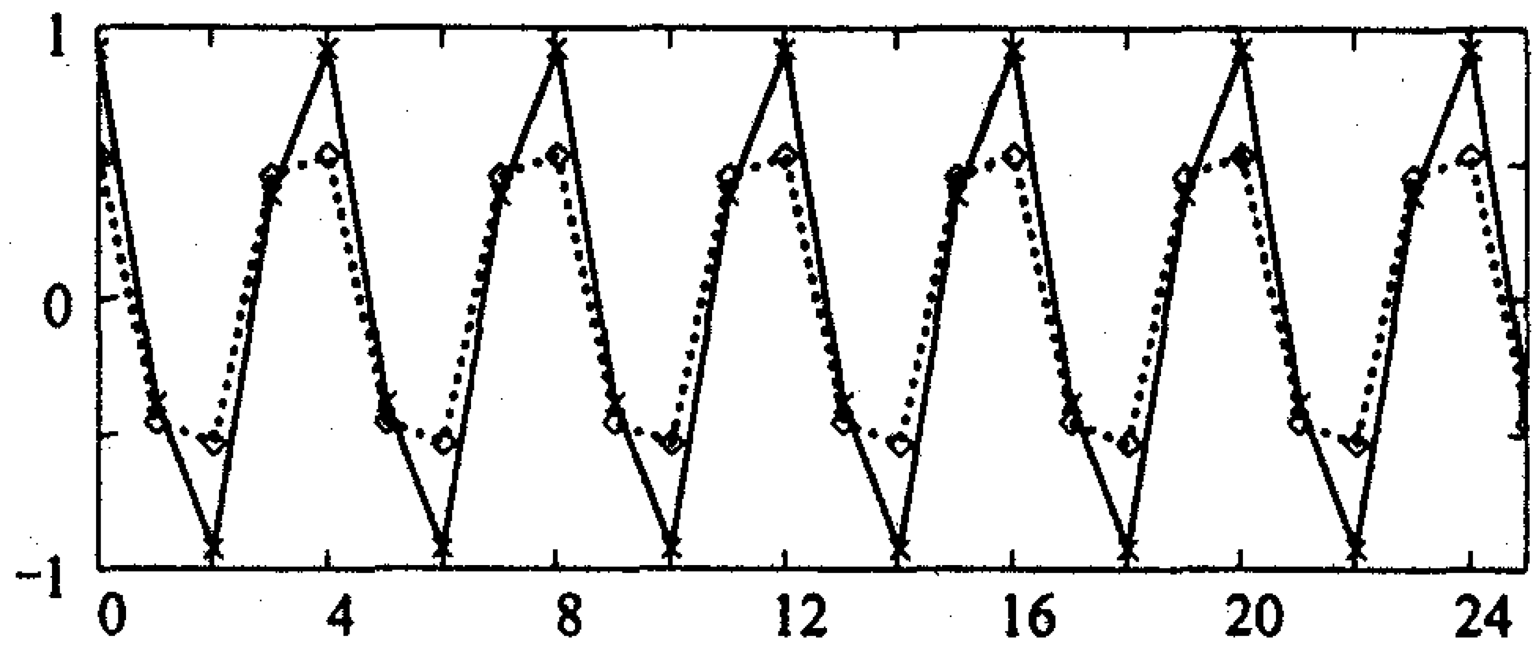

Fig. 2

shows the effect of the phase spread of the orthogonal channels ($A = 1$, $\varphi = -\pi/8$, $\Delta\psi = \arctan(U^S/U^C) - \varphi$), while the second one shows the effect of the amplitude nonidentity ($A = 1$, $\varphi = -\pi/8$), without taking into account the intrinsic noise of the receivers. The rows denoted by the symbols U^S and U^C correspond to quadrature components of output voltages in digital filters of additional gating [3], the rows marked by U correspond to their modules, and ΔU to relative errors of these modules.

Figure 2 shows the graphs of samples, picked by the ADC, of an ideal and undistorted sinusoidal signal (solid line), and that with quadrature errors (dotted line)

These results help us notice the following: (a) the phase quadrature error results in amplitude and phase distortion of the signal complex amplitude; (b) the absolute value of the phase distortion of the signal complex amplitude is twice less than the phase quadrature error, which can be explained by its redistribution over the orthogonal subchannels; (c) for the same reason, the absolute value of the amplitude distortion of the signal is twice less than the amplitude quadrature error; and (d) the amplitude quadrature error leads, by and large, to amplitude distortion of the resulting complex amplitude of the signal.

In order to eliminate the impact of the quadrature nonidentity of receivers' transfer factors on the output signal complex amplitude, we shall use a pair of two consecutive digital samples of quadrature subchannels. The first step on the way of realization of the correction method suggested consists in calculating the actual amplitude and phase errors by ADC time samples. As a result, we obtain the necessary calculation formulas permitting the evaluation of phase (Δ) and the amplitude ($\Delta A^* = 1 + \Delta A$) discrepancies.

To do this, let us rewrite (2) for the ith and (i + 1)-th time samples:

$$a_k(i) = a_s \sin(2\pi f_s i T_{\mathrm{ADC}} + \varphi_s), \tag{4}$$

$$a_k(i+1) = a_s \sin(2\pi f_s i T_{\mathrm{ADC}} + 2\pi f_s T_{\mathrm{ADC}} + \varphi_s). \tag{5}$$

With regard for (1) and assuming that $n = 0$, $T_0 = 1/f_0$, we obtain $2\pi f_s\, T_{\mathrm{ADC}} = \pi/2$, and rewrite (5) in the form

$$a_k(i+1) = a_c \sin(2\pi f_c i T_{adc} + \varphi_c + \pi/2) = a_c \cos(2\pi f_c i T_{adc} + \varphi_c). \tag{6}$$

By analogy, we can obtain the expressions for voltages at the output of the second subchannel for the ith and (i + 1)-th time samples:

$$b_k(i) = \Delta A^* a_s \sin(2\pi f_s i T_{\mathrm{ADC}} + \varphi_s + (\Delta + \pi/2)), \tag{7}$$

$$b_k(i+1) = \Delta A^* a_s \sin(2\pi f_s i T_{\mathrm{ADC}} + 2\pi f_s T_{\mathrm{ADC}} + (\Delta + \pi/2) + \varphi_s) = \Delta A^* a_s \cos(2\pi f_s i T_{\mathrm{ADC}} + (\Delta + \pi/2) + \varphi_s). \tag{8}$$

Now let us square expressions (4), (6)–(8) for the consecutive time samples of the orthogonal subchannels, and determine the pairwise sums of the squared time samples:

$$a_k^2(i) + a_k^2(i+1) = a_s^2(\sin^2(2\pi f_s i T_{\mathrm{ADC}} + \varphi_s) + \cos^2(2\pi f_s i T_{\mathrm{ADC}} + \varphi_s)) = a_s^2, \tag{9}$$

$$\begin{aligned} b_k^2(i) + b_k^2(i+1) = (\Delta A^*)^2(\sin^2(2\pi f_s i T_{\mathrm{ADC}} + (\Delta + \pi/2) + \varphi_s) + \\ + \cos^2(2\pi f_s i T_{\mathrm{ADC}} + (\Delta + \pi/2) + \varphi_s)) = (\Delta A^*)^2 a_s^2. \end{aligned} \tag{10}$$

After dividing (9) by (10), we come to the amplitude nonidentity sought for:

$$\Delta A^* = \sqrt{\left(b_k^2(i)+b_k^2(i+1)\right)/\left(a_k^2(i)+a_k^2(i+1)\right)}\,.$$

Now let us define the value of phase error Δ. With the use of trigonometric identities for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$, represent expressions (7) and (8) in the form

$$b_k(i)=\Delta A^* a_s \sin((2\pi f_s iT_{\text{ADC}}+\varphi_s+\pi/2)+\Delta)=\Delta A^* a_s(\sin(2\pi f_s iT_{\text{ADC}}+\varphi_s+\pi/2)\cos\Delta+$$
$$+\cos(2\pi f_s iT_{\text{ADC}}+\varphi_s+\pi/2)\sin\Delta)=\Delta A^*(a_k(i+1)\cos\Delta-a_k(i)\sin\Delta).$$

Setting $\cos\Delta=0$, we can write the final expression:

$$b_k(i)=\Delta A^*(a_k(i+1)-a_k(i)\tan\Delta)/\cos\Delta \tag{11}$$

By analogy, $b_k(i+1)$ can be written as

$$b_k(i+1)=\Delta A^* a_c\cos((2\pi f_c iT_{adc}+\varphi_c+\pi/2)+\Delta)=$$
$$=\Delta A^* a_s(\cos(2\pi f_s iT_{\text{ADC}}+\varphi_s+\pi/2)\cos\Delta-\sin(2\pi f_s iT_{\text{ADC}}+\varphi_s+\pi/2)\sin\Delta)=$$
$$=\Delta A^*(a_k(i)\cos\Delta+a_k(i+1)\sin\Delta)$$

or

$$b_k(i+1)=-\Delta A^*(a_k(i)+a_k(i+1)\tan\Delta)/\cos\Delta \tag{12}$$

Division of (11) by (12) yields the equation

$$\frac{b_k(i)}{b_k(i+1)}=-\frac{a_k(i+1)-a_k(i)\tan\Delta}{a_k(i)+a_k(i+1)\tan\Delta}=\frac{a_k(i)\tan\Delta-a_k(i+1)}{a_k(i)+a_k(i+1)\tan\Delta},$$

which, being resolved in terms of $\tan\Delta$, gives us

$$b_k(i)a_k(i)+b_k(i+1)a_k(i+1)=\tan\Delta(b_k(i+1)a_k(i)-b_k(i)a_k(i+1)), \tag{13}$$

from which

$$\Delta=\arctan((b_k(i)a_k(i)+b_k(i+1)a_k(i+1))/(b_k(i+1)a_k(i)-b_k(i)a_k(i+1)).$$

In order to average the estimates of the quadrature nonidentities over a totality of I samplings, we can use the least square method. Based on equations (9) and (10), let us write the residual function to determine the unknown ΔA^*:

$$F=\sum_{i=0}^{I-1}\left\{b_k^2(i)+b_k^2(i+1)-(\Delta A^*)^2\left(a_k^2(i)+a_k^2(i+1)\right)\right\}^2=\min.$$

By setting the derivative of F with respect to $(\Delta A^*)^2$ equal to zero, we find

$$\Delta A^*=\sqrt{\sum_{i=0}^{I-1}(b_k^2(i)+b_k^2(i+1))(a_k^2(i)+a_k^2(i+1))/\sum_{i=0}^{I-1}(a_k^2(i)+a_k^2(i+1))^2}\,.$$

This variant is the statistically optimal alternative to the trivial averaging

$$\Delta A^*=\sqrt{\sum_{i=0}^{I-1}(b_k^2(i)+b_k^2(i+1))/\sum_{i=0}^{I-1}(a_k^2(i)+a_k^2(i+1))}\,.$$

Based on (13), let us set up the residual function to estimate $\tan\Delta$:

$$F=\sum_{i=0}^{I-1}\{b_k(i)a_k(i)+b_k(i+1)a_k(i+1)-(b_k(i+1)a_k(i)-b_k(i)a_k(i+1))\tan\Delta\}=\min.$$

Table 3

$\Delta°$	–18	–9	–4.5	0	4.5	9	18
Δ^{*} °	–18	–9	–4.5	0	4.5	9	18

Table 4

$(1+\Delta A)$	0.7	0.8	0.9	0	1.1	1.2	1.3
ΔA^{*}	0.7	0.8	0.9	0	1.1	1.2	1.3

By resolving the equation $\partial F/\partial \tan\Delta = 0$ we can determine

$$\tan\Delta = \frac{\sum_{i=0}^{I-1}\left(a_k(i)\cdot b_k(i)+a_k(i+1)\cdot b_k(i+1)\right)\left(a_k(i)\cdot b_k(i+1)-a_k(i+1)\cdot b_k(i)\right)}{\sum_{i=0}^{I-1}\left(a_k(i)\cdot b_k(i+1)-a_k(i+1)\cdot b_k(i)\right)^2}$$

and then find Δ.

Analysis of the above formulas points to the fact that the estimates of the phase nonidentities of the orthogonal reception subchannels are invariant to amplitude nonidentities and vice versa. The performance check results of the algorithms for estimating the quadrature nonidentities of the orthogonal reception subchannels with the aid of the MathCad package in the absence of intrinsic noise are presented in Table 3 (estimation of the phase spread Δ^{*}) and in Table 4 (estimation of the phase spread ΔA^{*}).

Based on these estimates of quadrature nonidentities, we suggest performing, in real time scale, the appropriate correction of time samples coming from ADC to the input of the next digital segment of the digital channel. Since the algorithm for estimating the quadrature nonidentities has been derived under the assumption that the transfer factor of only one reception subchannel is changing, it is reasonable to correct time samples in the same subchannel for every clock period of the ADC.

To set up the correction algorithm, represent the current *i*th sample (3) of the distorted quadrature subchannel in the form

$$b_k(i) = \Delta A^{*} a_s(\sin(2\pi f_s iT_{\mathrm{ADC}} + \varphi_s + \pi/2)\cos\Delta + \cos(2\pi f_s iT_{\mathrm{ADC}} + \varphi_s + \pi/2)\sin\Delta) =$$

$$= \Delta A^{*}(b_{k,\mathrm{cor}}(i)\cos\Delta - \sin(2\pi f_s iT_{\mathrm{ADC}} + \varphi_s)\sin\Delta) = \Delta A^{*} b_{k,\mathrm{cor}}(i)\cos\Delta - \Delta A^{*} a_k(i)\sin\Delta$$

Consequently, $b_{k,\mathrm{cor}}(i)\Delta A^{*}\cos\Delta = b_k(i) + \Delta A^{*} a_k(i)\sin\Delta$, and the algorithm of disbalance correction can be written as

$$b_{k,\mathrm{cor}}(i) = (b_k(i) + \Delta A^{*} a_k(i)\sin\Delta)/(\Delta A^{*}\cos\Delta) = a_k(i)\tan\Delta + b_k(i)/(\Delta A^{*}\cos\Delta),$$

where $a_k(i)$ and $b_k(i)$ are time samples of the ADC at the output of the orthogonal reception subchannels; while Δ and ΔA^{*} are estimates of the phase error of "dequadratization" and of the absolute value of amplitude distortion.

The testing of the synthesized algorithm of correction was performed with the aid of calculations implemented in the MathCad medium using the above-mentioned procedures of signal processing. The results of correction of quadrature nonidentities of orthogonal subchannels, in conformity with the algorithm suggested, and in the absence of receivers' intrinsic noise are presented in Table 5 (correction of phase nonidentities, $A = 1$, $\varphi = -\pi/8$, $\Delta\psi = \arctan(U^S/U^C) - \varphi$) and Table 6

Table 5

$\Delta°$	−18	−9	−4.5	0	4.5	9	18
U^C	29.564	29.564	29.564	29.564	29.564	29.564	29.564
U^S	−12.246	−12.246	−12.246	−12.246	−12.246	−12.246	−12.246
U	32	32	32	32	32	32	32
ΔU,%	0	0	0	0	0	0	0
$\Delta\psi$	0	−4.5	0	0	0	0	0

Table 6

$(1 + \Delta A)$	0.7	0.8	0.9	1	1.1	1.2	1.3
ΔA,%	−30	−20	−10	0	10	20	30
U^C	29.564	29.564	29.564	29.564	29.564	29.564	29.564
U^S	−12.246	−12.246	−12.246	−12.246	−12.246	−12.246	−12.246
U	32	32	32	32	32	32	32
ΔU,%	0	0	0	0	0	0	0
$\Delta\psi$	0	0	0	0	0	0	0

(correction of amplitude nonidentities, $A = 1$, $\varphi = -\pi/8$). The comparison between Tables 1, 2 and 5, 6 points to the efficiency of the new algorithms for correction of quadrature disbalance of the channels.

In order to diminish the impact of intrinsic noise of the reception path on the accuracy of the correcting coefficients, one has to apply a harmonic pilot signal, whose intensity exceeds the noise variance by 20–25 dB, and to use its coherent accumulation.

Thus the suggested method of correction makes it possible to soften the requirements for the identity of the quadrature subchannels, and to accuracy of the 90° rotation of the phase at the analog orthogonalization of signals in the receiving channel of digital antenna arrays.

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